Waves or particles?

- Light travels in straight lines
  - Waves travel in circles (chuck a rock in a pond and watch the ripples spread out)
  - But particles in crossed beams would collide?
- Light reflects off mirrors and leaves at the same angle as it came in
  - Makes sense for particles (conservation of momentum)

Waves or particles?

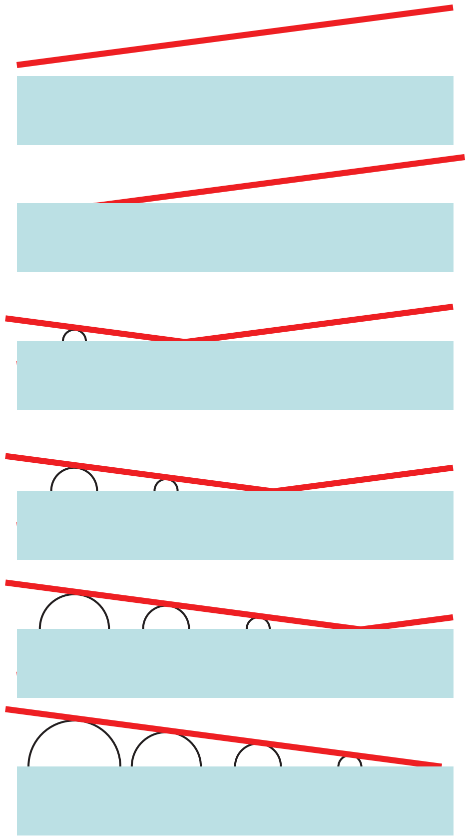
- Light bends (refracts) when moving between different media

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

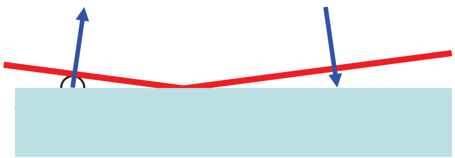
  - Newton had a semi-plausible explanation for particles
  - Easy to explain for waves *if* they travel in straight lines!

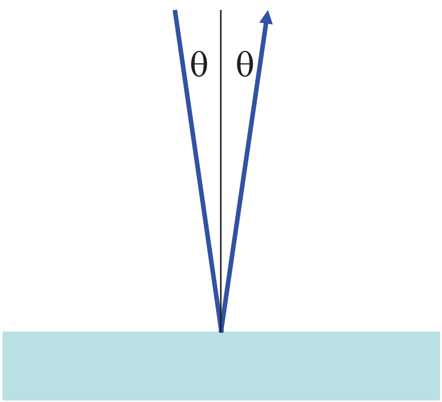
Reflection

- Wavefront propagates in a straight line
- As it hits the surface it becomes a source of secondary wavelets
- Wavelets all “grow” at the same speed
- Envelope of these forms new wavefront



Reflection

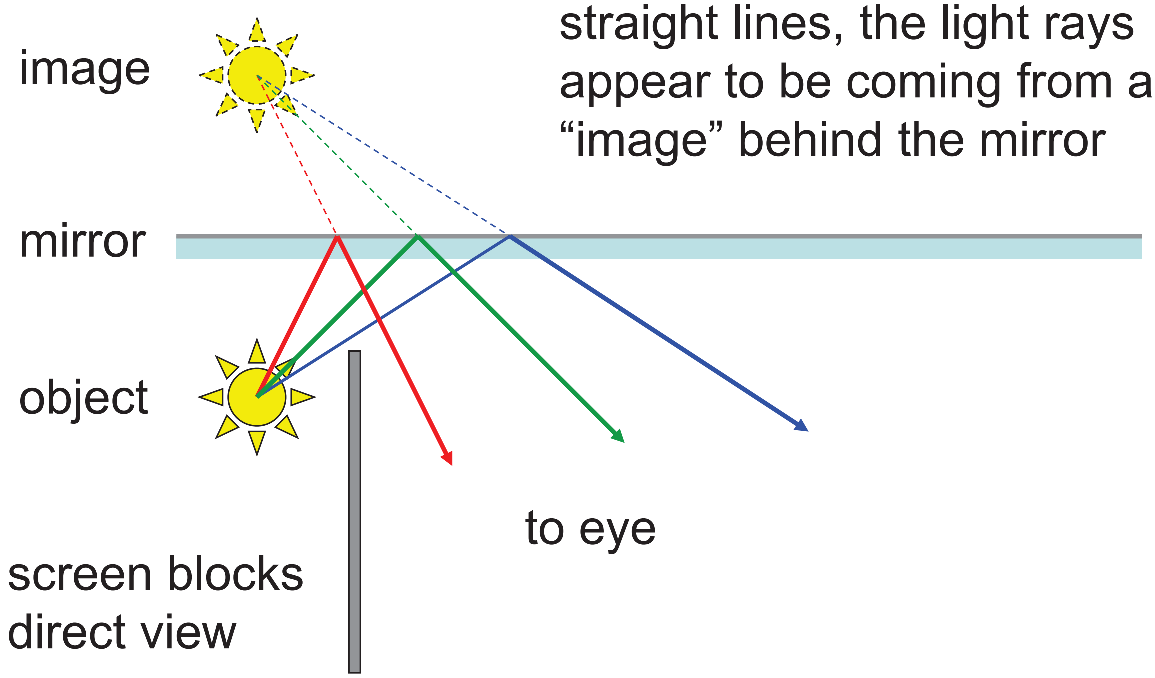




- Reflected ray is at the same angle as incident ray
- Reflected wavefront is at the same angle as incident wavefront
- Occurs because the secondary wavelets grow at the same rate in both wavefronts

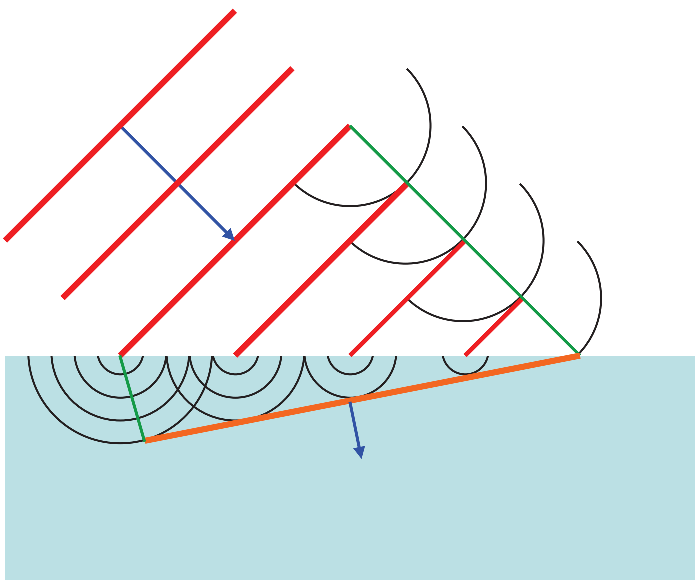
Image in a mirror

Since light normally travels in straight lines, the light rays appear to be coming from an "image" behind the mirror



Refraction

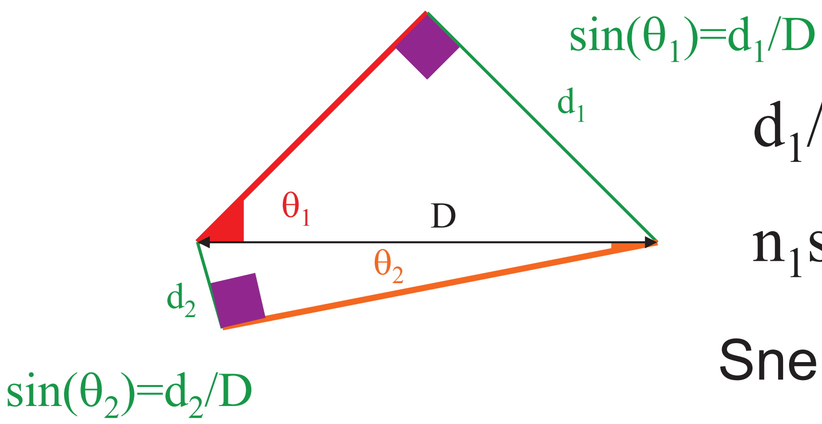
- Refraction is easily explained if wavelets travel more slowly in glass than in air



The two green lines are both four wavelets long. The start points of each line are points on a wavefront and so the end points must also be corresponding points on the new wavefront

Refraction

- The light ray takes the same length of time to travel along the two green paths
- Travels at different speeds: $v=c/n$, where n is the refractive index



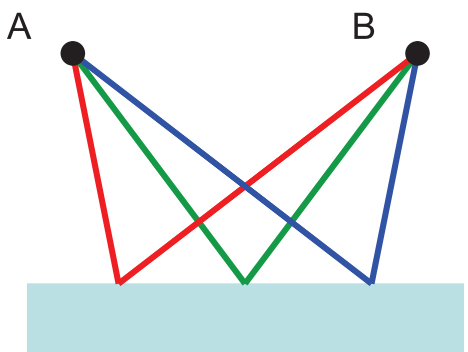
$$d_1/v_1 = d_2/v_2$$

$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$

Snell's law of refraction

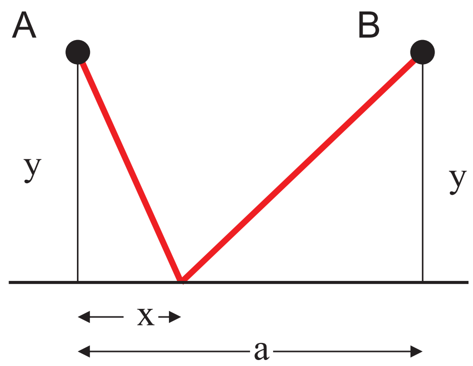
Reflection (Fermat)

A light ray takes the shortest (*least time*) path between two points



- At constant speed least time is equivalent to shortest distance
- Consistent with light moving in straight lines
- The green line is shorter than the red and blue lines
- Shortest path between A and B via the mirror!

Reflection (Fermat)



- Need to minimise total distance

$$s = \sqrt{y^2 + x^2} + \sqrt{y^2 + (a-x)^2}$$

$$\frac{ds}{dx} = \tfrac{1}{2}\left(y^2 + x^2\right)^{-1/2} \times 2x + \tfrac{1}{2}\left(y^2 + (a-x)^2\right)^{-1/2} \times 2(a-x)(-1) = 0$$

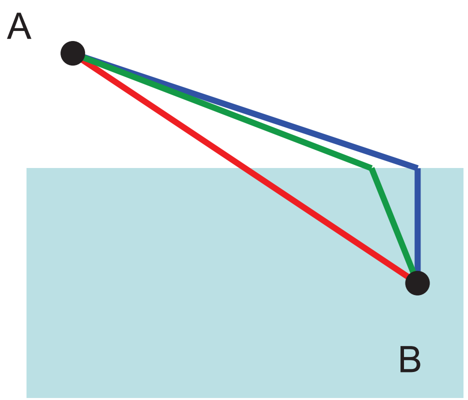
$$x = a - x$$

$$x = a/2$$

- Or use geometrical insight to spot that the answer is obvious if you reflect point B in the mirror.

Refraction (Fermat)

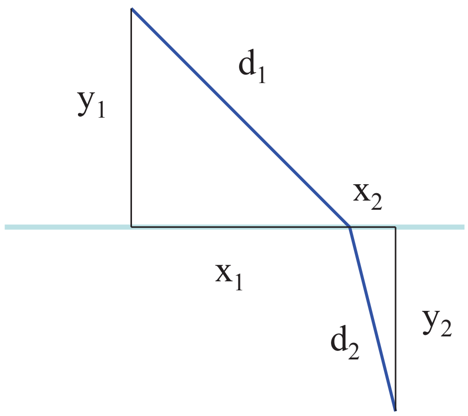
A light ray takes the shortest (*least time*) path between two points



- At varying speed least time is *not* equivalent to shortest distance
- Light moves in straight lines in one medium but will bend at joins
- The green line is the *quickest* path between A and B!

Refraction (Fermat)

- Minimise total time taken to travel along path

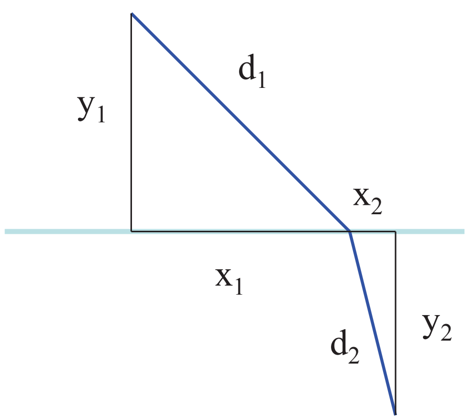


$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c}$$

$$= \frac{n_1\sqrt{x_1^2 + y_1^2} + n_2\sqrt{x_2^2 + y_2^2}}{c}$$

Solve $dt/dx_1=0$

Refraction (Fermat)



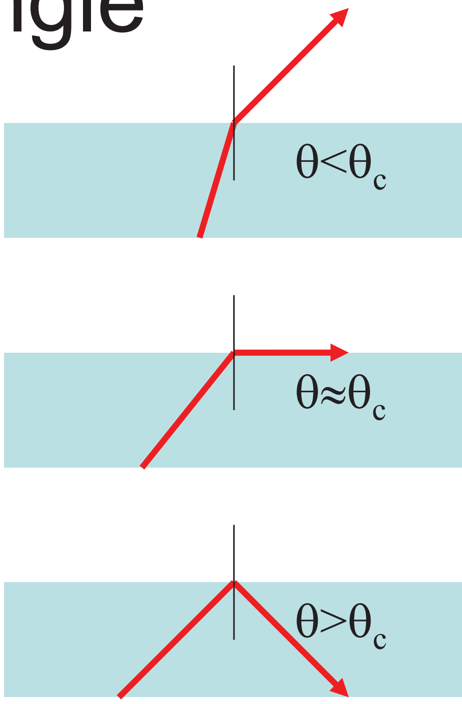
$$\frac{\mathrm{d}t}{\mathrm{d}x_1} = \frac{n_1}{c} \times \tfrac{1}{2}\left(x_1^2 + y_1^2\right)^{-\frac{1}{2}} \times 2x_1$$

$$+ \frac{n_2}{c} \times \tfrac{1}{2}\left(x_2^2 + y_2^2\right)^{-\frac{1}{2}} \times 2x_2 \times (-1)$$

$$\frac{\mathrm{d}t}{\mathrm{d}x_1} = \frac{n_1 x_1}{c d_1} - \frac{n_2 x_2}{c d_2}$$

$$= \frac{n_1 \sin(\theta_1)}{c} - \frac{n_2 \sin(\theta_2)}{c}$$

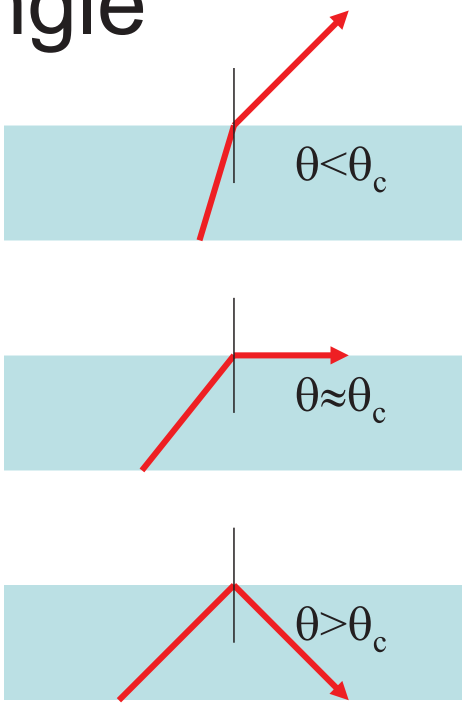
Critical angle



- A light ray travelling from a material with high refractive index to one with low refractive index is always bent away from the normal
- Angle is limited to 90º

Beyond the critical angle light ray undergoes *total internal reflection*

Critical angle



$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$

At $\theta_1 = \theta_c$, $\theta_2 = 90^o$

$$\sin(\theta_c) = n_2/n_1$$

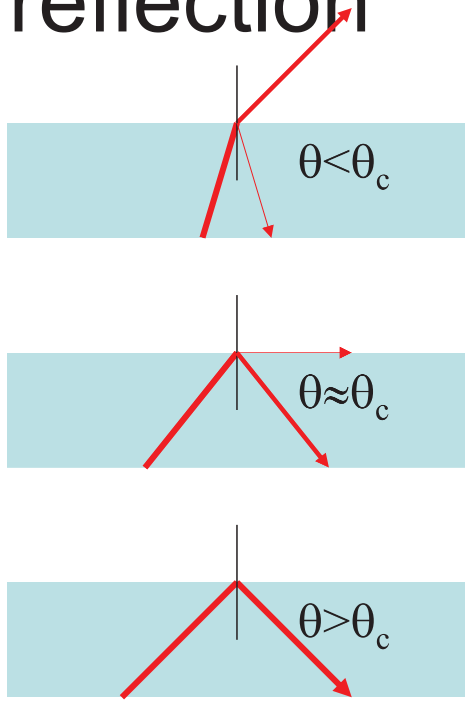
For glass to air

$$\theta_c \approx \sin^{-1}(1/n)$$

Beyond the critical angle light ray undergoes *total internal reflection*

Partial internal reflection

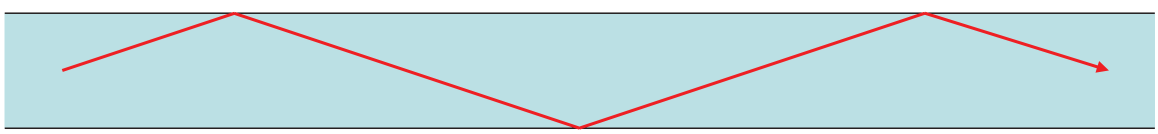
- For all angles less that the critical angle there is both a transmitted ray *and* a reflected ray
- Beyond critical angle light ray undergoes *total internal reflection*



The reflected ray is always reflected at the incident angle

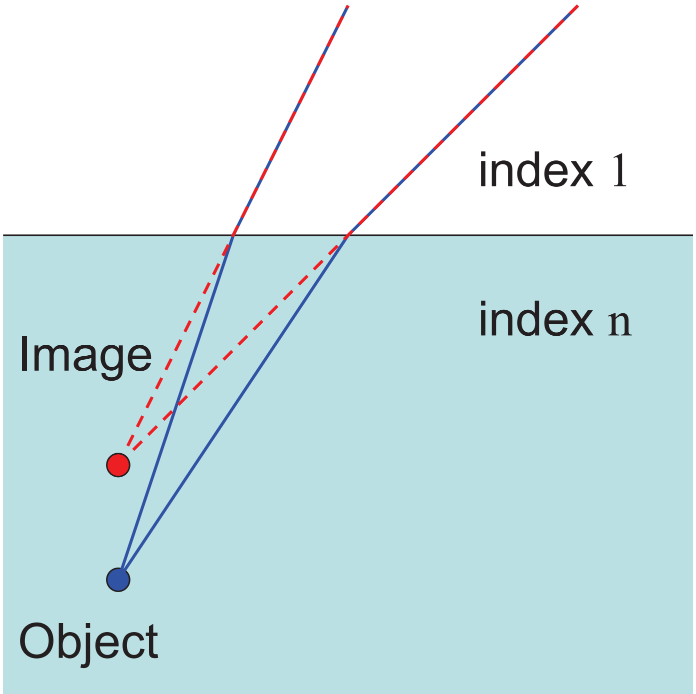
Optic fibres (light pipes)

- Light can travel along an optic fibre by a series of total internal reflections
- If first reflection is beyond the critical angle then all reflections will be; the limit of transmission is set by the transparency of the glass



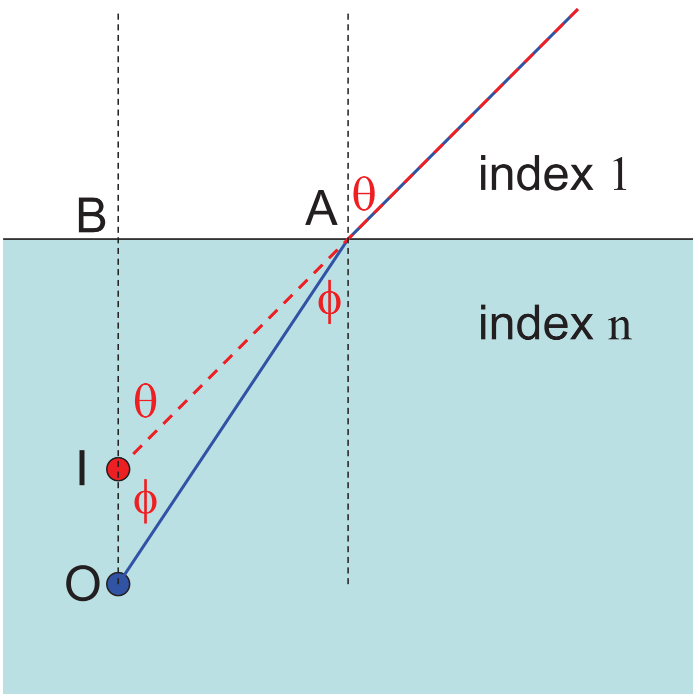
- Real fibres are made from two sorts of glass

Real and apparent depth



- If an underwater object is viewed from above it will appear to be in a different place from where it really is
- More on images later!
- Apparent depth is reduced by a factor of the refractive index *n*

Real and apparent depth



$n = \sin(\theta)/\sin(\phi)$
$\approx \tan(\theta)/\tan(\phi)$

$\tan(\theta) = AB/IB$
$\tan(\phi) = AB/OB$

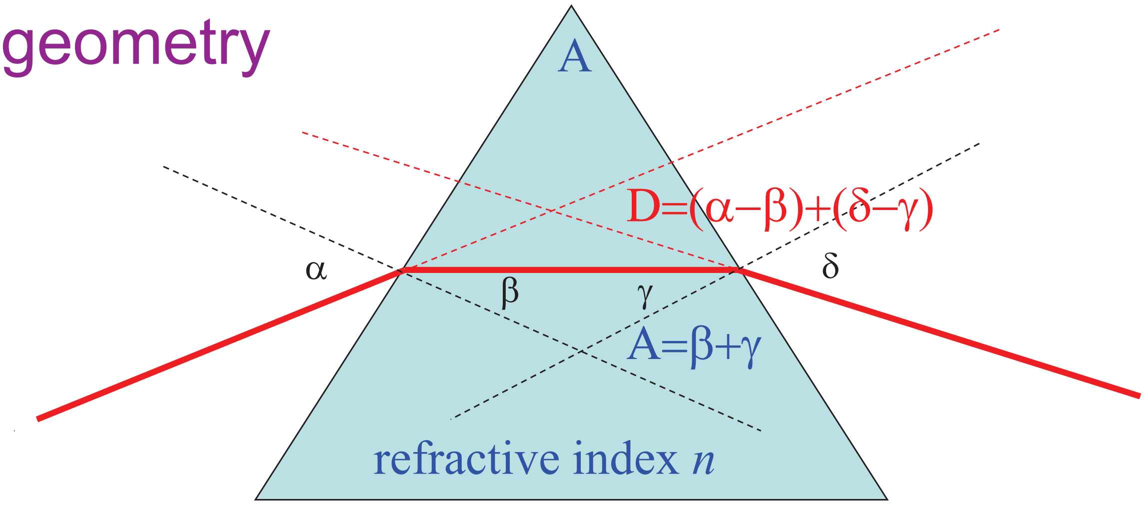
$n = (AB/IB)/(AB/OB)$

$= OB/IB$

All rays appear to come from point I at depth OB/n

Refraction at a prism

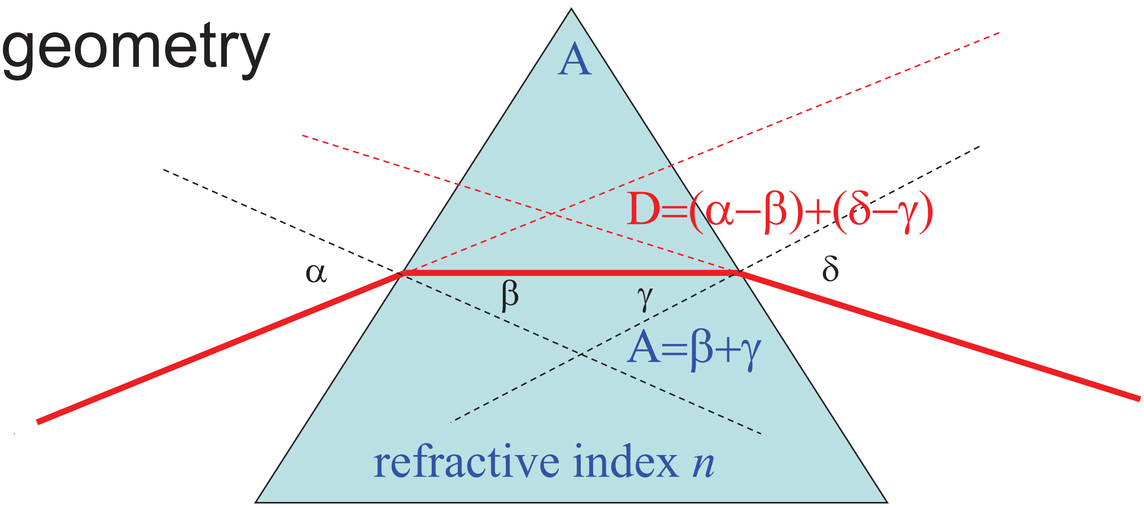
1) geometry



2) optics $\sin(\alpha)=n\times\sin(\beta)$ $\sin(\delta)=n\times\sin(\gamma)$

Small angles

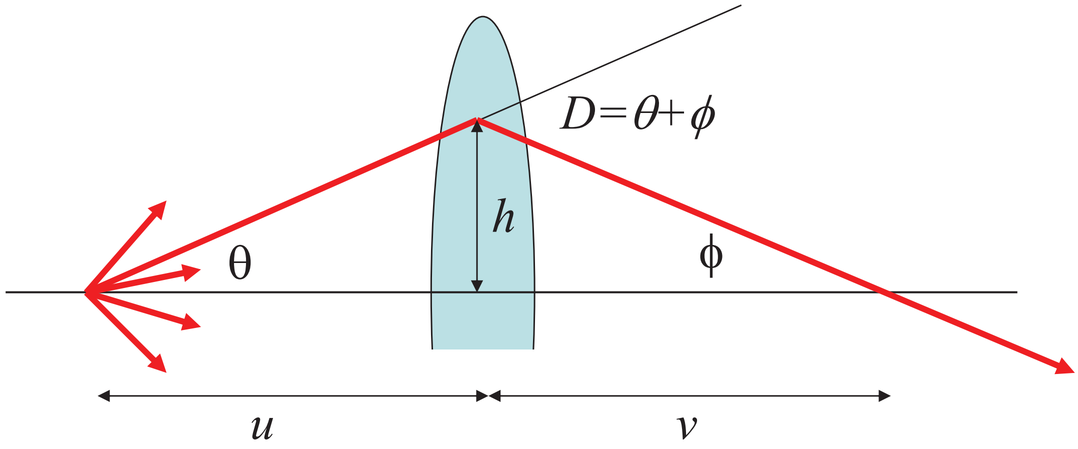
1) geometry



2) optics $\alpha=n\times\beta$ $\delta=n\times\gamma$

$$D=(n-1)\times\beta+(n-1)\times\gamma=(n-1)\times(\beta+\gamma)=(n-1)\times A$$

The lens formula (1)



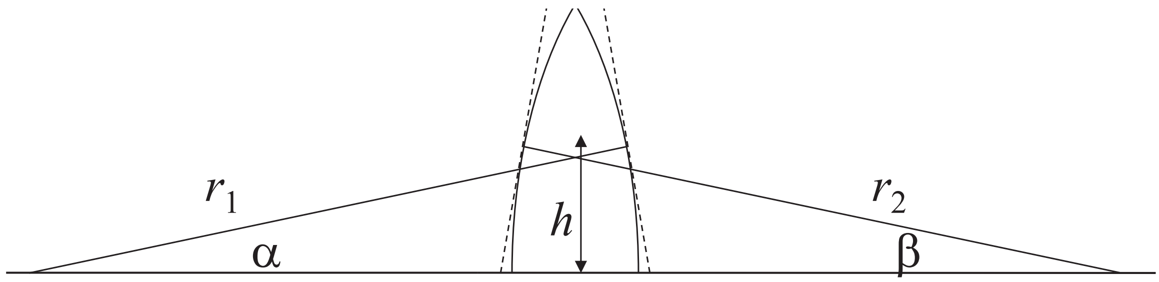
$$D = \theta + \phi = (n-1)\times A$$

rays focussed if

$$\theta \approx h/u \quad \phi \approx h/v$$

$$A \approx h/C$$

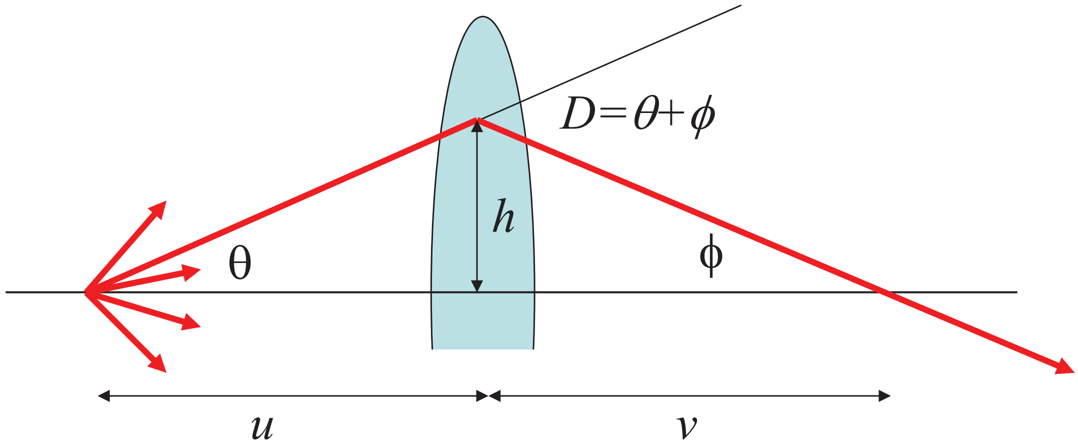
The lens formula (2)



A lens is formed by a pair of curved surfaces. The angle of the equivalent prism is the angle between the surface tangents, which equals the sum of α and β.

For spherical surfaces $\alpha \approx h/r_1$ and $\beta \approx h/r_2$ where r_1 and r_2 are the radii of the two spheres

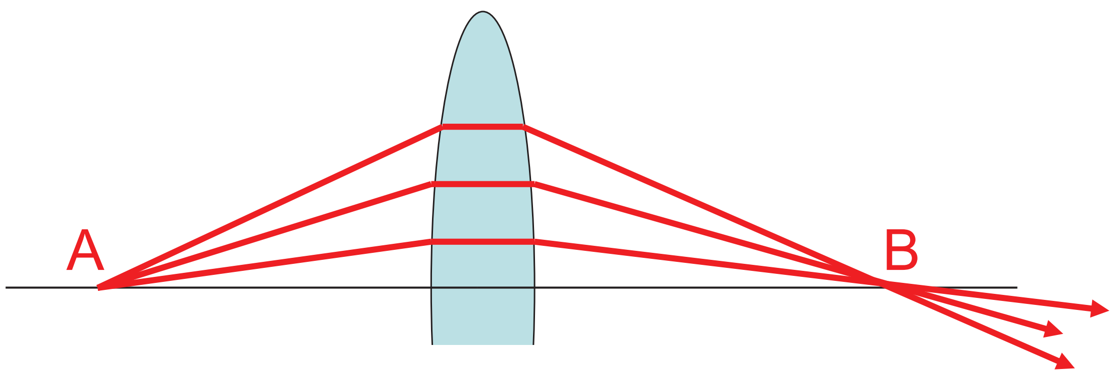
The lens formula (3)



$$h/u + h/v = (n-1) \times (h/r_1 + h/r_2)$$

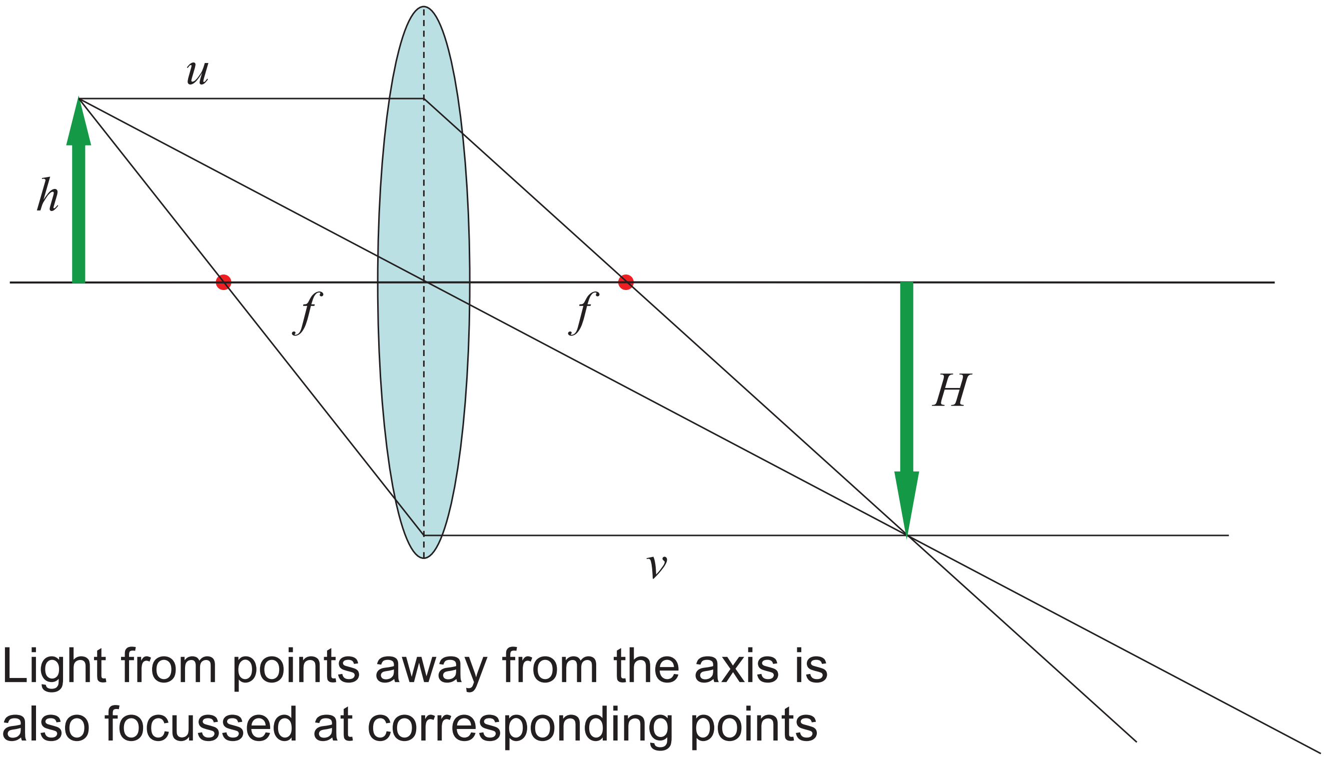
$$1/u + 1/v = (n-1) \times (1/r_1 + 1/r_2) = 1/f$$

A lens (Fermat)



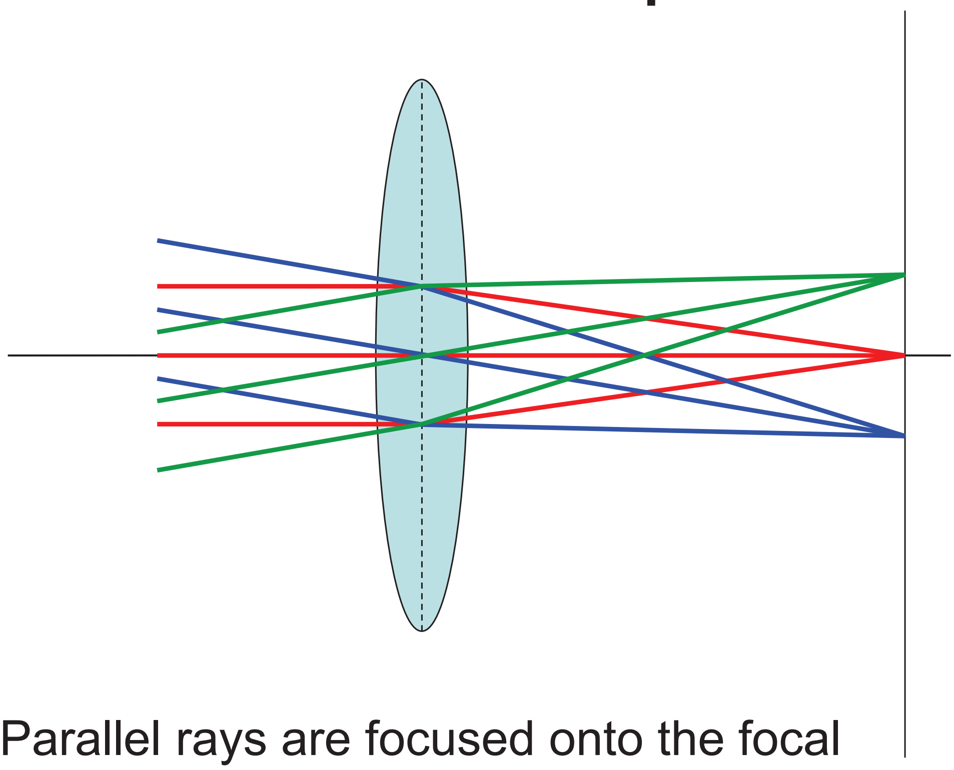
- Light can take several different paths from A to B
- All paths must be minimum time, so all must take the same time! Lens must be shaped so that extra length in air cancels shorter length in glass

Extended objects



Light from points away from the axis is also focussed at corresponding points

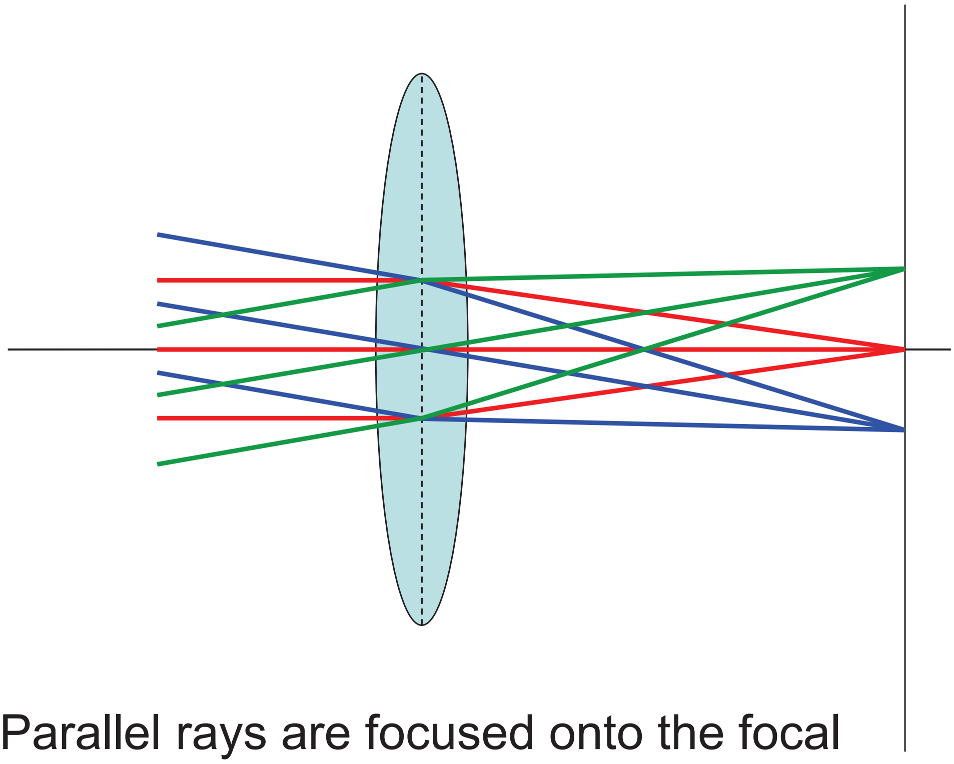
Focal plane



The lens converts angles to positions!

Parallel rays are focused onto the focal plane: in the limit $u \to \infty$ then $v \to f$

Landscape camera



Place film or a CCD detector in the focal plane. If the object is not at infinity then must move lens away from detector or decrease its focal length

Parallel rays are focused onto the focal plane: in the limit $u \to \infty$ then $v \to f$

Magnification of real image

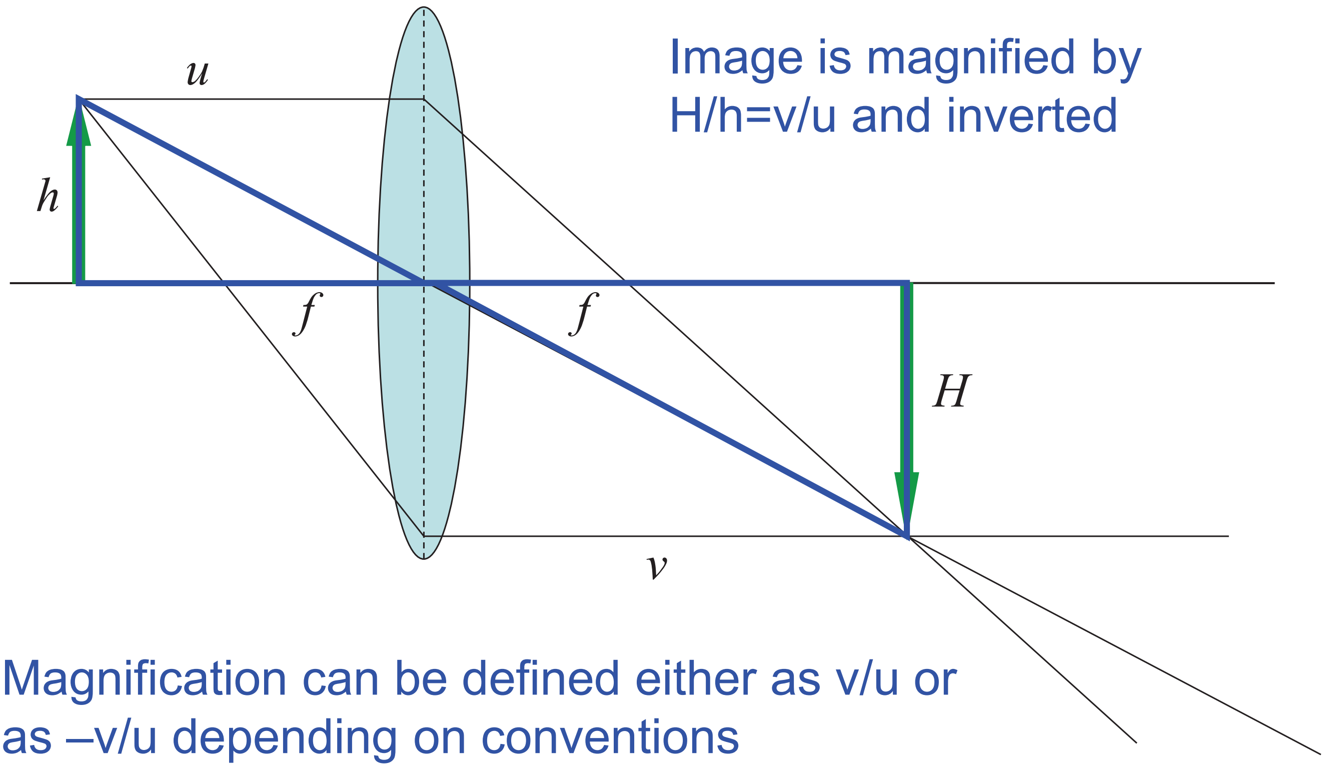
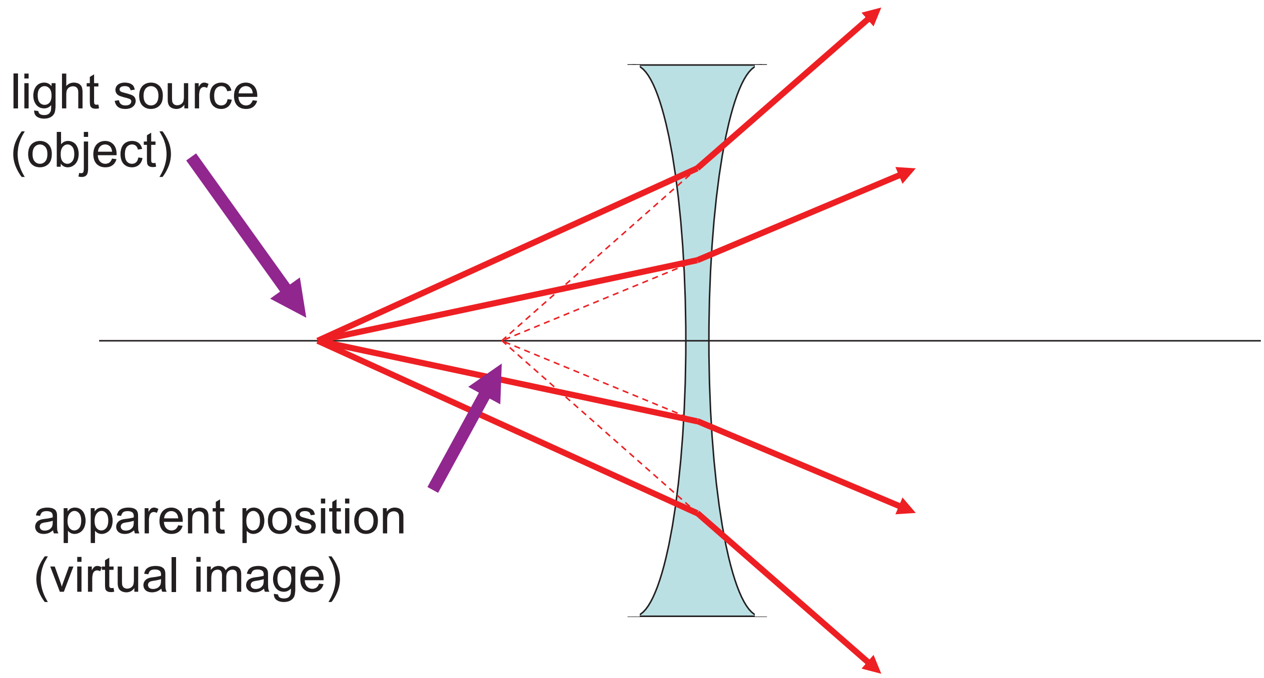


Image is magnified by H/h=v/u and inverted

Magnification can be defined either as v/u or as –v/u depending on conventions

Virtual image with a lens



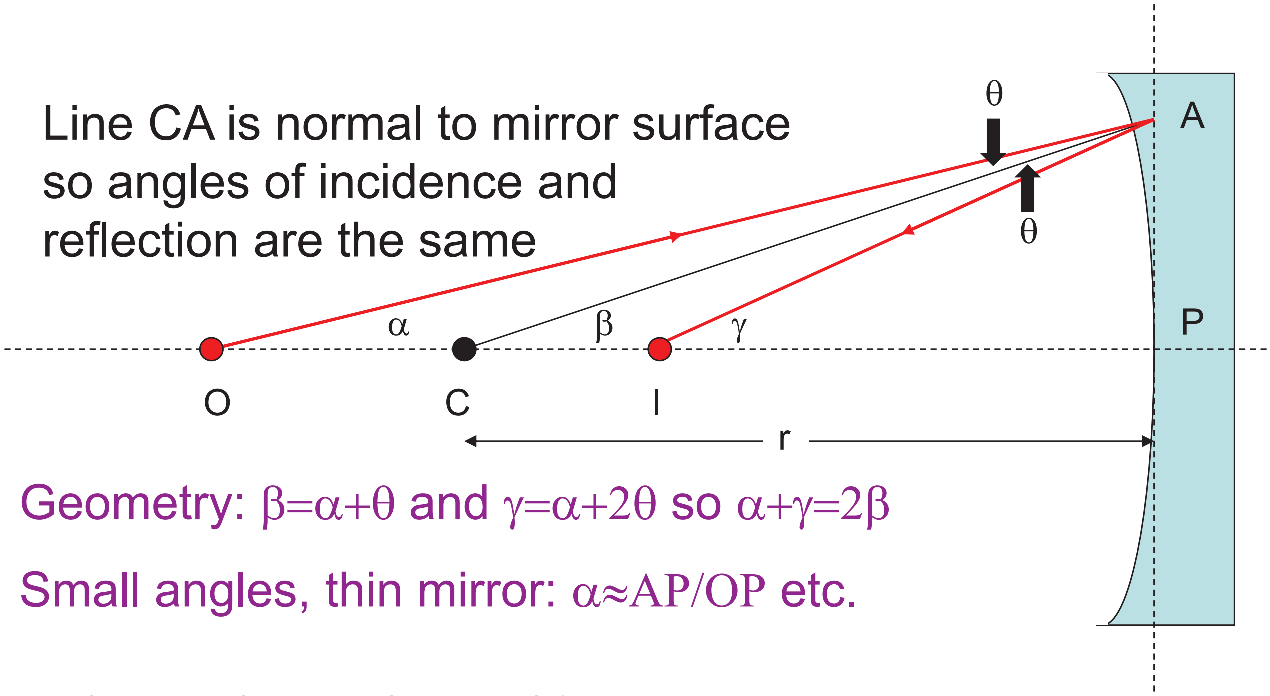
Virtual images of extended objects are scaled down by v/u and upright (draw a ray diagram to check)

Lens summary

Object	Image type	Location	Orientation	Size
Converging Lens (f>0)				
∞>u>2f	Real	f<v<2f	Inverted	Reduced
u=2f	Real	v=2f	Inverted	Same size
2f>u>f	Real	2f<v<∞	Inverted	Magnified
u=f	Beam	±∞		
u<f	Virtual		Erect	Magnified
Diverging lens (f<0)				
Anywhere	Virtual	f<v<0	Erect	Reduced

The mirror formula

Line CA is normal to mirror surface so angles of incidence and reflection are the same



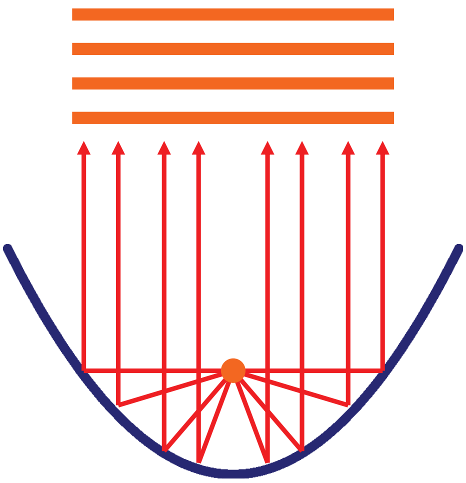
Geometry: $\beta=\alpha+\theta$ and $\gamma=\alpha+2\theta$ so $\alpha+\gamma=2\beta$

Small angles, thin mirror: $\alpha \approx AP/OP$ etc.

$$1/u + 1/v = 2/r = 1/f$$

Parabolic Mirrors (2)

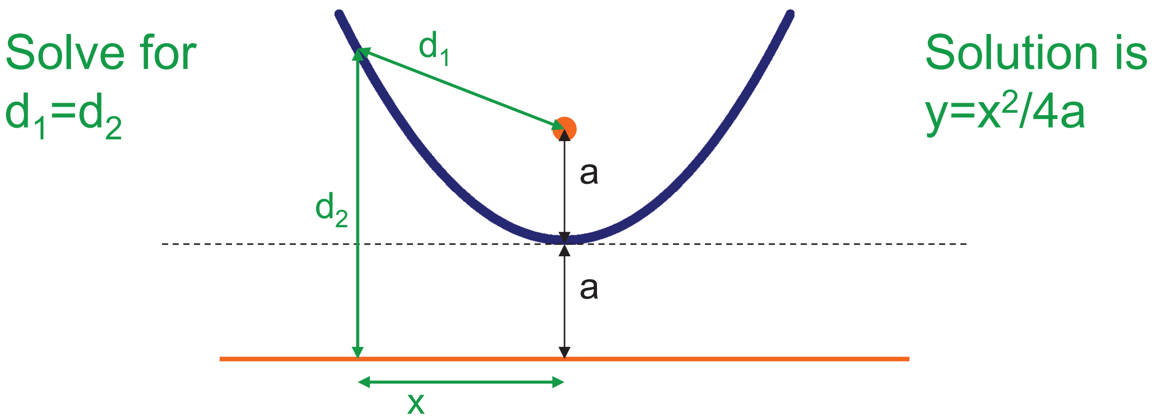
- We can deduce the shape needed from Fermat's principle or from straight wavefronts



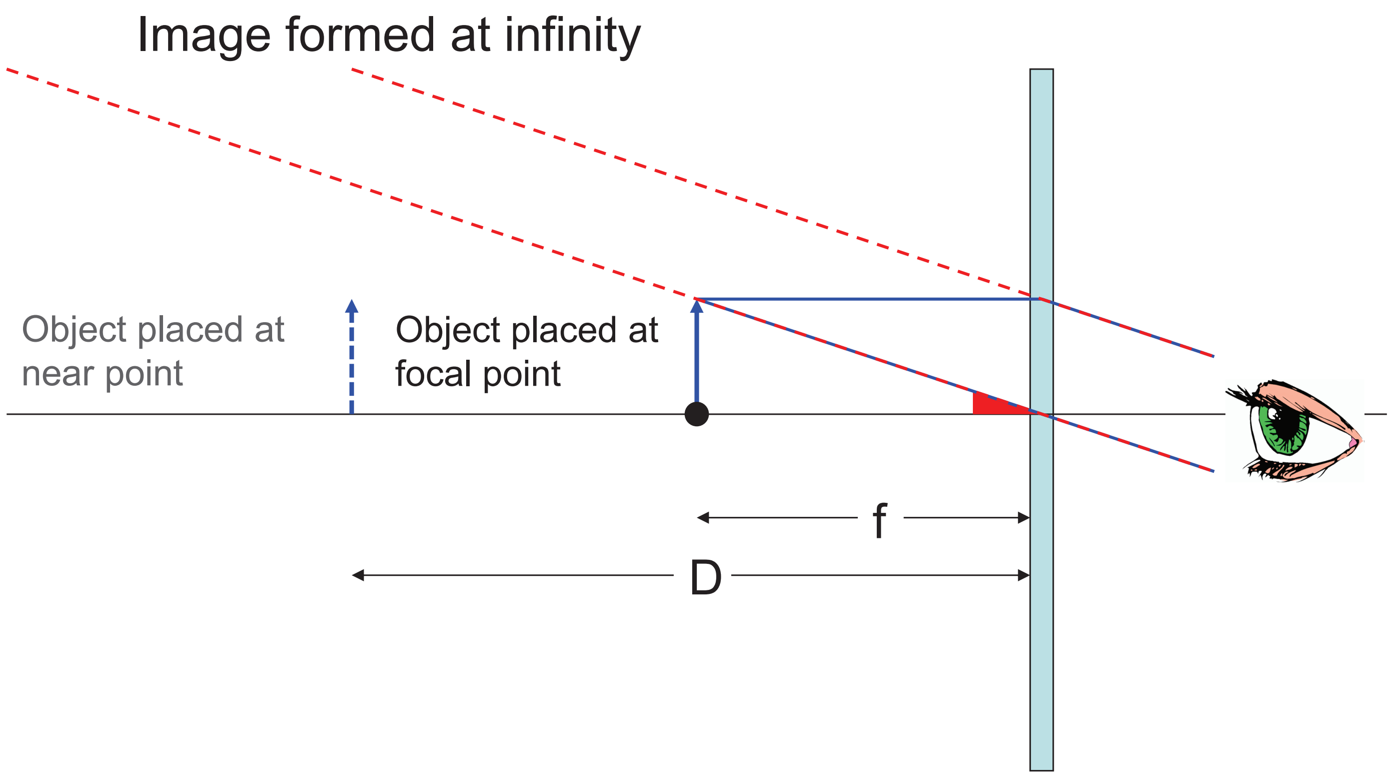
Place a source as indicated at the "focus" of the mirror. This will produce plane wavefronts if the red lines all have the same length

Parabolic Mirrors (3)

- Geometric definition of a parabola: the locus of all points equidistant from a line and a point at a distance 2a from the line (the focus)



Magnifying glass (4)



Magnifying glass (6)

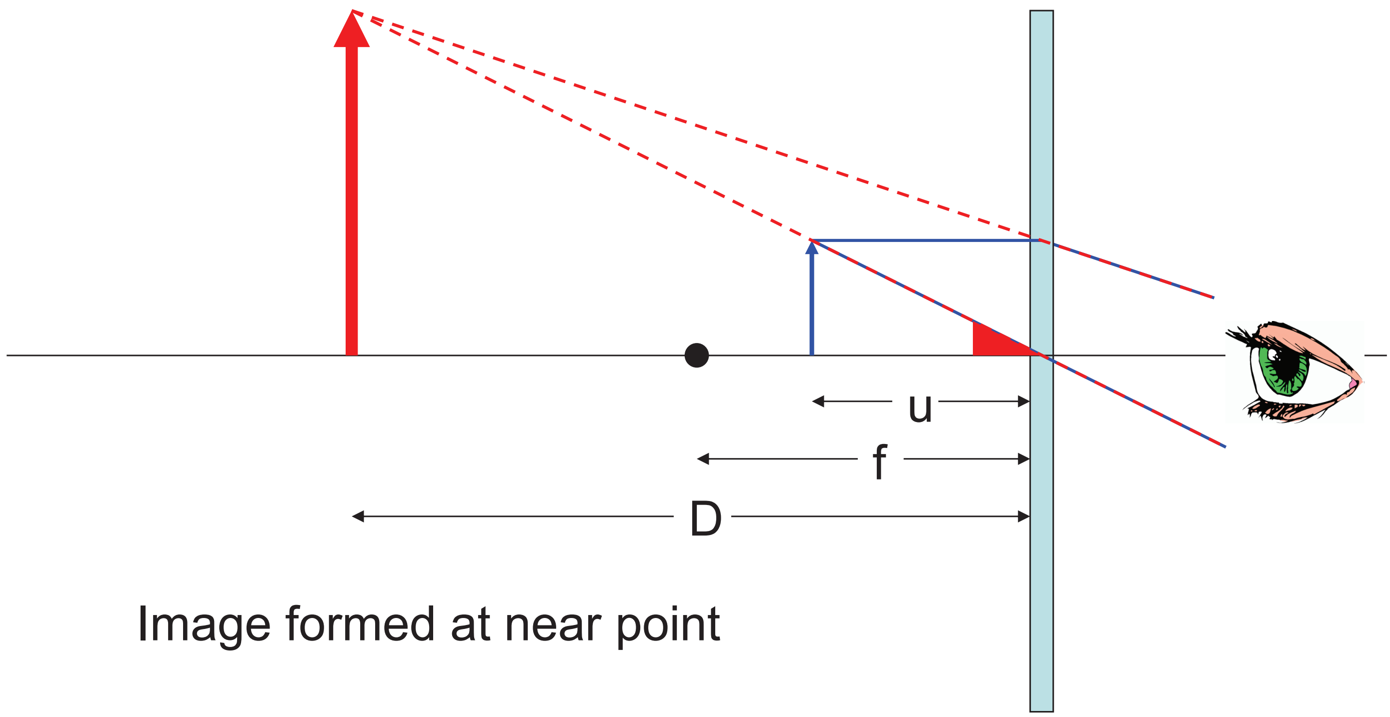
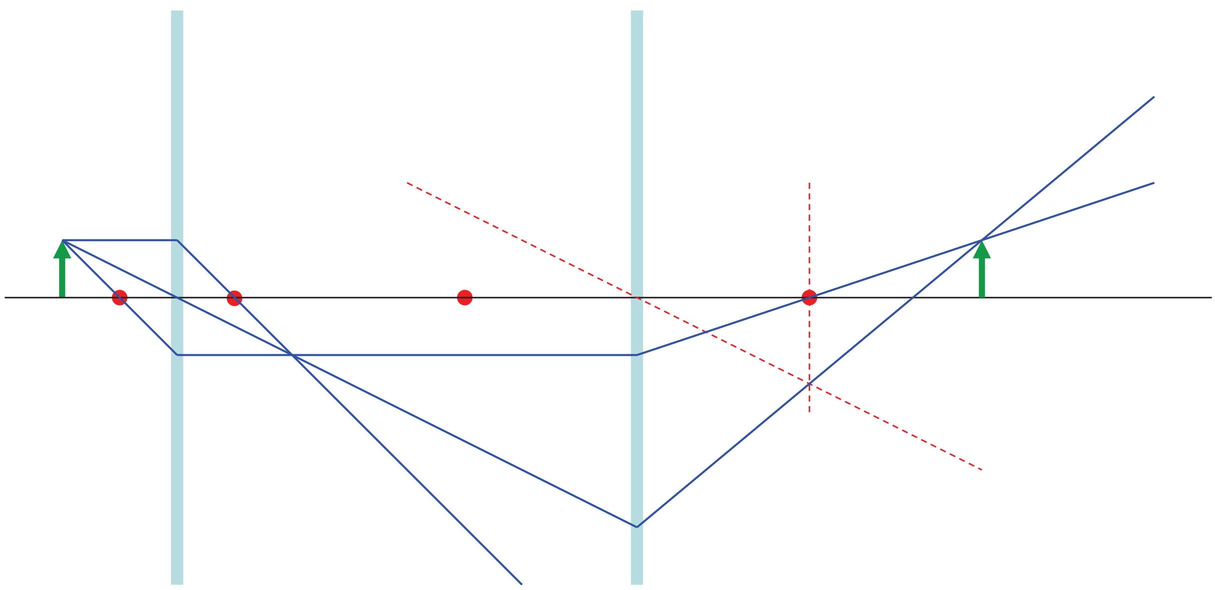


Image formed at near point

Ray tracing with two lenses (2)



Can ray trace in complex systems by constructing “assistant rays” parallel to the ray of interest and passing through the centre of a lens