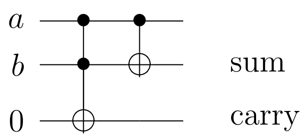
A Simple networks and algorithms

1. Show that for reversible classical computing CLONE and SWAP gates can be built out of networks of controlled-NOT gates. Can these networks also be used for quantum computing? Show how to build NOT and controlled-NOT gates from Toffoli gates. Design a reversible OR gate using only Toffoli gates and NOT gates.

2. Show how a Fredkin gate can be used to implement reversible AND and NOT gates.

3. Consider the reversible half adder network



   Explain how this network works. Why isn't it necessary to preserve the second input?

4. Given an oracle U_f implementing a function from one bit to one bit, design a classical circuit to directly determine the parity of f in two queries using only two bits (that is you may not store results "offline" for later comparisons and you may not use additional ancilla bits).

5. The notes give explicit circuits corresponding to the four functions f_{ij} in Deutsch's algorithm. Find an alternative circuit for f_{10} which does not apply any single qubit gates to the upper qubit. Hence or otherwise use circuit identities to convert the circuit for Deutsch's algorithm in the case of f_{10} to the form given in the notes.

6. Consider a Deutsch–Jozsa problem with $n = 2$: how many possible functions are there, and how many are constant and how many are balanced? Assuming that an unknown function is known to be either constant or balanced with 50% probability, calculate the minimum, maximum, and average number of queries required to determine which sort of function it is on a classical computer. What about a quantum computer?

7. Calculate an explicit matrix form for the Grover amplitude amplification operator in the case $n = 2$, and hence show that Grover's quantum search will reveal a single satisfying input in a single query (you may neglect the ancilla qubit and use an explicit phase shift form for the action of the controlled gate on the two main qubits). What happens if the function has *two* satisfying inputs? What about three?

B Error correction and decoherence

1. Show that the two encoding networks for quantum error correction given in the notes will act as desired, and write down corresponding decoding networks.

2. Consider the three qubit spin flip error correcting network shown in the notes. By working through the network, find kets describing the state of the device immediately before the ancilla qubits are measured for an arbitrary logical input with each of the three single qubit errors

or no error. Show that these states can be written as product states of the logical qubit and the ancilla qubits, and hence show that measuring the ancillas has no effect on the logical qubit.

3. Give explicit forms for the error correcting steps in the three qubit spin flip error correcting network (that is, what correction operators should be applied for each correction outcome). Show how this process can be replaced by quantum control (replacing measurements and optional gates by conditional logic gates). State two disadvantages of this latter approach.

4. What happens to a classical bit protected with a three bit code if two bit flip errors occur? What happens in the quantum case?

5. Consider a single qubit undergoing two sorts of decoherence: (a) a rotation around the z-axis through an angle of $\pm\phi$ chosen at random; (b) experiencing a Z gate with probability p. Show that the density matrix description of these two situations is fundamentally equivalent, and determine the relationship between p and ϕ.

C Atoms and ions

1. Write down the potential energy function for a group of ions (each of mass M and charge $+e$) in a linear Paul trap, with strong radial and weak axial confinement. You may assume that the trap potentials are harmonic. What effect does the motion of an ion have on its spectral lines if the ions is travelling in free space? What changes if the ion is confined in a harmonic trap? Outline briefly how sideband cooling works for trapped ions.

2. The ${}^{40}\mathrm{Ca}^{+}$ ion trap uses a transition at 729 nm, while the ${}^{9}\mathrm{Be}^{+}$ ion trap works at 1.25 GHz. Calculate the spontaneous decay time of a strongly allowed transition at these energies, using $1/\Gamma = (3\pi\epsilon_0\hbar c^3)/(\omega^3 e^2 z^2)$, where $z \sim a_0$. How does the ${}^{40}\mathrm{Ca}^{+}$ ion trap avoid rapid relaxation? How does the ${}^{9}\mathrm{Be}^{+}$ ion trap achieve spatial discrimination?

3. Draw a quantum network based on the the collisional phase gate U_ϕ described in the lecture notes to implement a controlled-NOT gate with the first qubit as control and the second qubit as target.

4. Show that the collisional phase gate U_π can be written as $|0\rangle\langle 0| \otimes \mathrm{Z} + |1\rangle\langle 1| \otimes \mathbb{1}$. Hence show that the "massive entanglement" state of a system of two atoms can be written as

$$(|0\rangle \mathrm{Z} + |1\rangle)\,(|0\rangle + |1\rangle)$$

neglecting normalisation. How would you write the state for three atoms? Multiply this out to show that you agree with the form given in the notes.

D Finals questions

1. Answer question 5 from the paper at `http://nmr.physics.ox.ac.uk/teaching/c2mock.pdf`

2. Answer question 5 from the 2006 C2 paper. This is easier than it might look!