Quantum information processing: Implementations

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Suggested reading

Lecturer material

All the material for this lecture can be found on the internet at the the following address:

http://users.ox.ac.uk/˜ clar0085/04HTQIP.html

Bibliography

- Quantum Computation and Quantum Information, by Michael A. Nielsen and Isaac L. Chuang, Chapter 7.
- Quantum optics, by D.F. Walls and G.J. Milburn, Chapter 10.

Contents

1 Introduction

In previous lectures you have seen how simple quantum systems like a qubit with only two basis states $|0\rangle$ and $|1\rangle$ can be used to perform quantum information processing (QIP) by applying unitary transformations $\mathcal{U}$ to it. It is not obvious that there are realistic physical systems which – under certain conditions – behave like a qubit and allow to perform quantum gates. In the following two lectures we will look at atoms in electromagnetic fields and show that this can indeed be achieved. We start by considering an atom interacting with a laser field and afterwards consider the case of nuclear spins in a magnetic field.

2 Atom in a laser field

In this section we look at the time evolution of an atom that interacts with a classical laser field. Our treatment will neglect any effect of spontaneous emission, i.e. we will only consider the coherent part of the time evolution. In the next section we will then discuss under which situations such a treatment can be valid.

2.1 Hamiltonian of an atom

We first consider the Hamiltonian H_A of an atom ignoring any interaction with an electromagnetic field. The eigenstates of the atom are labelled by $|n\rangle$. Although it is not possible to analytically calculate these eigenstates for atoms more complicated than hydrogen their properties like e.g. the energy or the symmetries of the corresponding wave functions are well known for a variety of atoms from experiments, simple approximations, and numerical calculations (see lectures on atomic physics). We will not attempt to calculate them but instead assume that they are known and we can thus simply write (in units $\hbar = 1$)

$$H_A |n\rangle = \omega_n |n\rangle \tag{1}$$

with a given value for ω_n. As H_A is Hermitian its eigenstates form a basis and we can therefore express the wave function $|\Psi\rangle$ of the atom in terms of this basis

$$|\Psi(t)\rangle = \sum_n c_n(t) |n\rangle \tag{2}$$

where c_n are found to be $c_n = \langle n| \Psi\rangle$. [1] In the previous lectures on QIP this system Hamiltonian H_A was referred to as the *background Hamiltonian.*

2.2 Interaction of the atom with a classical laser field

Next we consider a classical laser field with frequency ω and wave vector $\mathbf{k}$. The electric field at time t and position $\mathbf{x}$ is given by

$$\mathbf{E}(\mathbf{x}, t) = \mathcal{E}(t)\epsilon e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})} + \text{c.c} \tag{3}$$

where ϵ is the polarization vector and $\mathcal{E}(t)$ is a slowly varying amplitude (which we will assume to be almost constant in time). At optical frequencies the interaction between the atom and the magnetic field of the laser is negligible and we therefore only have to determine the effects

[1] Note that strictly speaking the above sum should also contain the continuous part of the spectrum describing ionized states which we do not indicate separately here for simplicity.

of the electrical field on the atom. Since the size of the atom will typically be of the order of the Bohr radius a_0 while the typical wave length of a laser is several hundred nanometers we can assume the electric field to be constant over the size of the atom which we assume to be trapped (more on trapping in Sec. 5) at the position $\mathbf{x} = 0$. The interaction energy of atom and laser in dipole approximation is given by $\mu \cdot \mathbf{E}(\mathbf{0}, t)$ with μ the atomic dipole which leads to the interaction Hamiltonian

$$H_{\text{int}} = \hat{\mu} \cdot \mathbf{E}(\mathbf{0}, t) \tag{4}$$

where now $\hat{\mu}$ is the atomic dipole operator (see problem 1 for a simple example on hydrogen like atoms). The Hamiltonian H_{int} describes the laser control field interacting with the atom.

2.3 Time evolution of the atom in the laser field

Let us assume that the frequency of the laser ω is close to the transition frequency between ground and first excited state, i.e. $\omega \approx \omega_1 - \omega_0$ and very different from any other transition frequency. With the dipole matrix elements $\mu_{nm} = \langle n | \hat{\mu} \cdot \epsilon | m \rangle$ we find for the time evolution of the atoms under the Hamiltonian $H = H_A + H_{\text{int}}$ using the time dependent Schrödinger equation $i |\Psi\rangle = H |\Psi\rangle$ and Eq. (2)

$$i \frac{dc_k}{dt} = \omega_k c_k - \left(\sum_n \mu_{kn} \mathcal{E}(t) e^{-i\omega t} c_n(t) + \text{c.c.} \right). \tag{5}$$

We now assume that the atom is initially in its ground state $|\Psi(t = 0)\rangle = |0\rangle$, i.e. $c_n = \delta_{n0}$ and make a Floquet ansatz for the coefficients $c_n = \sum_{l=-\infty}^{\infty} c_n^{(l)} e^{-i(\omega_0 + l\omega)t}$ with initial condition $c_n^{(l)} = \delta_{n0}\delta_{l0}$. We thus find

$$i c_k^{(l)}(t) = (\omega_k - \omega_0 - l\omega) c_k^{(l)}(t) - \sum_n \mu_{kn} \mathcal{E}(t) c_k^{(l-1)}(t) - \sum_n \mu_{nk} \mathcal{E}^*(t) c_k^{(l+1)}(t) \tag{6}$$

From this equation we see that almost every coefficient will be oscillating very quickly with a frequency difference $\delta_{lk} = l\omega - (\omega_k - \omega_0)$ and only the two coefficients $c_0^{(0)}$ and $c_1^{(1)}$, where according to our assumption of $\omega \approx \omega_1 - \omega_0$ the quantity δ_{lk} is small, will be varying slowly in time. The quickly oscillating contributions to the above sums in Eq. (6) will average out over the longer time scales relevant for $c_0^{(0)}$ and $c_1^{(1)}$. Therefore we are led to the two level approximation (considering only populations in the two levels $|0\rangle$ and $|1\rangle$) and the so called rotating wave approximation (neglecting all terms oscillating on a short (optical) time scale). Applying these approximations to the above Eq. (6) we find

$$\begin{aligned} i c_0^{(0)} &= -\Omega^*(t) c_1^{(1)} \\ i c_1^{(1)} &= -\Delta c_1^{(1)} - \Omega(t) c_0^{(0)}. \end{aligned} \tag{7}$$

where $\Delta = \delta_{11}$ is called the detuning, it is the difference between the laser frequency and the atomic transition frequency. Furthermore we have defined the Rabi-frequency $\Omega(t) = 2\mu_{10}\mathcal{E}(t)$ which is proportional to the strength of the electric field of the laser light.

2.4 Allowed and forbidden transitions

The above analysis shows that under a number of assumptions on the laser parameters and properties of the atom the whole dynamics of the atom is described by only taking two of the

eigenstates into account. In a realistic atom there are lots of eigenstates which quite often lie close together (see problem 3) However, even in this situation it is often possible to use a laser to selectively interact with only two levels as a number of transitions will be dipole forbidden. For a forbidden transition between $|n\rangle$ and $|k\rangle$ the dipole matrix element μ_{kn} is zero and thus there will be no contribution to the above sums of Eq. (6) even if this transition is near resonant.

2.5 Two level atom in the laser field

We can now proceed by introducing a reduced (time dependent) basis consisting only of the two states $|\tilde{0}\rangle = |0\rangle$ and $|\tilde{1}\rangle = e^{-i\omega t}|1\rangle$. We use these two states as our qubit and leave out the $\tilde{}$ in the following. According to the above discussion the time evolution of the atom is then given by $|\Psi(t)\rangle = c_0^{(0)}(t)|0\rangle + c_1^{(1)}(t)|1\rangle$. In matrix form the evolution equation for the coefficients is given by

$$i\begin{pmatrix} \dot{c}_0^{(0)} \\ \dot{c}_1^{(1)} \end{pmatrix} = H_{\text{red}} \begin{pmatrix} c_0^{(0)} \\ c_1^{(1)} \end{pmatrix}. \tag{8}$$

which is a Schrödinger equation for a two level system with the reduced Hamiltonian

$$H_{\text{red}} = \begin{pmatrix} 0 & \Omega^*/2 \\ \Omega/2 & -\Delta \end{pmatrix}. \tag{9}$$

In Dirac notation this Hamiltonian is denoted as $H_{\text{red}} = -\Delta|1\rangle\langle 1| + (\Omega|1\rangle\langle 0| + \text{h.c.})$. We note that the parameters appearing in this Hamiltonian can be adjusted experimentally in a wide range as they are determined by external laser parameters.

3 Raman pulses

The above scheme for implementing single qubit operations in atoms brings along a number of problems we have gently overlooked until now. One of the most important ones is that the excited state $|1\rangle$ will spontaneously decay and as the transition from state $|1\rangle$ to $|0\rangle$ was assumed to be dipole allowed this will usually happen on a fast time scale typically on the order of ns and destroy the quantum information stored in the atom. Also we have chosen a time dependent basis for the qubit above which oscillates at an optical frequency. This large frequency will make it experimentally very hard to keep track of the phase of the excited state $|1\rangle$. Therefore we will now look at a different atomic configuration which does not suffer from these problems. It involves two classical laser beams and three of the atomic states.

3.1 The λ system driven by two lasers

We consider the situation where two laser beams with Rabi frequencies Ω_0 and Ω_1 drive transitions between the atomic states $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$, respectively. The states $|0\rangle$ and $|1\rangle$ are assumed to be chosen from the ground state manifold of the atom and the state $|e\rangle$ is an excited state. We denote the detunings of the two lasers from the atomic transitions by Δ_0 and Δ_1, respectively. In a realistic situation this can be achieved by making use of the selection rules and/or choice of frequencies. In a very similar way as above we find the Hamiltonian for this situation to be

$$H = -\Delta_R|e\rangle\langle e| + \left(\frac{\Omega_0}{2}|e\rangle\langle 0| + \text{h.c.}\right) + \left(\frac{\Omega_1}{2}|e\rangle\langle 1| + \text{h.c.}\right) - \delta|1\rangle\langle 1|, \tag{10}$$

where Δ_R is the detuning of the laser Ω_0 from the transition $|0\rangle \leftrightarrow |e\rangle$ and δ is the detuning of the Raman transition from the transition $|0\rangle \leftrightarrow |1\rangle$. The laser parameters are chosen such that $\Delta_R \gg \Omega_0, \Omega_1, \delta$.

3.2 Reduction to a two level system

With the above choice of parameters the excited state $|e\rangle$ will hardly be occupied and therefore we will try to eliminate it from the dynamics of our system. We write the wave function of the system (in the time dependent basis) as $|\Psi(t)\rangle = c_0(t)\,|0\rangle + c_1(t)\,|1\rangle + c_e(t)\,|e\rangle$ and find for the evolution equation for $c_e(t)$ from the Schrödinger equation

$$i\dot{c}_e(t) = -\Delta_R c_e(t) + (\Omega_0 c_0(t) + \Omega_1 c_1(t))/2. \tag{11}$$

The solution of this equation with initial condition $c_e(-\infty) = 0$ is given by

$$c_e(t) = -i\int_{-\infty}^{t} dt' e^{-i\Delta_R(t-t')}(\Omega_0 c_0(t') + \Omega_1 c_1(t'))/2. \tag{12}$$

Since Δ_R is much larger than any other frequency appearing in this problem the coefficient $c_e(t)$ will change very quickly compared to the other two coefficients. However, we do not set it to zero as we did before for quickly rotating coefficients, but use a more accurate approximation this time. We calculate the integral in Eq. (12) approximately by replacing $c_{0,1}$ with their values at time t. We therefore find $c_e(t) = (\Omega_0 c_0(t) + \Omega_1 c_1(t))/2\Delta_R$. Substituting this result into the equations for $c_0(t)$ and $c_1(t)$ we obtain a closed set of equations and have reduced the dynamics of the three level system to a two level system with Hamiltonian $H_{\rm red}$ where now $\Delta = \delta + (\Omega_0^2 - \Omega_1^2)/4\Delta_R$ and $\Omega = \Omega_0\Omega_1/4\Delta_R$. This reduced Hamiltonian causes transitions between two states $|0\rangle$ and $|1\rangle$ as before. However, there are a few points worth noting here:

- The transfer of population between two different metastable ground state levels by two classical laser fields is called Raman transition and $\pi/2$ or π pulses performed in this way are Raman pulses.
- Both states $|0\rangle$ and $|1\rangle$ are from the ground state manifold, so there is no spontaneous decay from these levels and therefore quantum information can be stored more reliably in these states.
- The population in state $|e\rangle$ remains small at all times and (for $\Omega_0 \approx \Omega_1$) is approximately given by (Ω/Δ_R). From atomic physics we know that the probability $p_s(dt)$ of a spontaneous decay of the atom during a small time dt is given by $p_s(dt) = A dt \Omega/\Delta_R$ with A the Einstein A-coefficient. During a gate time with $dt = \pi/\Omega$ the probability $p_s(dt) = A\pi/\Delta_R$ can be kept small when choosing sufficiently large values of Δ_R.
- The energy difference between the states $|0\rangle$ and $|1\rangle$ is much smaller than in the case of an optical transition between the states $|0\rangle$ and $|1\rangle$. It is thus easier to follow the phase of the rotating basis state in the case of a Raman transition.
- Due to the requirement $\Delta_R \gg \Omega_1, \Omega_2$ the resulting Rabi frequency Ω will usually be smaller than for directly driven transitions.

4 Rabi flopping as quantum gates

The time evolution due to $H_{\rm red}$ is often called Rabi flopping. We now show how some of the basic quantum gate operations can be implemented by this Hamiltonian if the two states $|0\rangle$ and $|1\rangle$ are considered a qubit.

4.1 Resonant case $\Delta = 0$

4.1.1 Rabi flopping

We look at the time evolution of the system for real values of Ω and find the time evolution operator $\mathcal{U}(t)$ (see the previous lectures on QIP)

$$\mathcal{U}(t) = e^{-iH_{\rm red}t} = \begin{pmatrix} \cos(\Omega t/2) & -i\sin(\Omega t/2) \\ -i\sin(\Omega t/2) & \cos(\Omega t/2) \end{pmatrix}. \tag{13}$$

Initially all the population of the system is in the state $|0\rangle$ and the wave function then evolves according to

$$|\Psi(t)\rangle = \cos(\Omega t/2)\,|0\rangle - i\sin(\Omega t/2)\,|1\rangle\,. \tag{14}$$

This yields oscillations in the populations of the two atomic states $p_n(t) = |\,\langle n|\,\Psi(t)\rangle\,|^2$ with a frequency of Ω the Rabi frequency.

$$\begin{aligned} p_0(t) &= (\cos(\Omega t/2))^2 = \frac{1}{2}\,(1+\cos(\Omega t)) \\ p_1(t) &= 1-(\cos(\Omega t/2))^2 = \frac{1}{2}\,(1-\cos(\Omega t))\,. \end{aligned} \tag{15}$$

This process is called Rabi flopping and by choosing different evolution times we can use it to implement the following basic quantum gates.

4.1.2 SQUARE ROOT OF NOT gate

We assume Ω to be real and write the Hamiltonian in spin notation where it is given by $H_{\rm red} = \Omega\sigma_x/2$. Applied for a time t with $\Omega t = \pi/2$ the induced time evolution is given by

$$e^{-iH_{\rm red}t} = e^{-i\pi/2\sigma_x/2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \tag{16}$$

which is the SQUARE ROOT OF NOT gate (often also called a $\pi/2$ pulse). Applying this Hamiltonian for a time t with $\Omega t = \pi$ yields a NOT gate (the global phase does not matter) with the propagator

$$e^{-iH_{\rm red}t} = e^{-i\pi\sigma_x/2} = -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{17}$$

in quantum optics also called π pulse.

4.1.3 Hadamard gate

If we assume Ω to be imaginary and apply $H_{\rm red}$ for a time t with $\Omega t = \pi/2$ we find the time evolution to be given by

$$e^{-iH_{\rm red}t} = e^{-i\pi/2\sigma_y/2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{18}$$

thus implementing a Hadamard gate.

4.2 Off resonant case $\Delta \neq 0$

4.2.1 Rabi flopping

We again calculate the time evolution operator $\mathcal{U}(t)$, define $s = \sqrt{\Delta^2 + \Omega^2}/2$ and find

$$\mathcal{U}(t) = e^{-iH_{\rm red}t/} = e^{i\Delta t/2} \begin{pmatrix} \cos(st) - i\Delta \sin(st)/2s & -i\Omega \sin(st)/2s \\ -i\Omega \sin(st)/2s & \cos(st) + i\Delta \sin(st)/2s \end{pmatrix}. \quad (19)$$

The populations for an atom initially in state $|0\rangle$ evolve in time according to

$$\begin{aligned} p_0(t) &= \cos(st)^2 + \frac{\Delta^2 \sin(st)^2}{4s^2} \\ p_1(t) &= \frac{\Omega^2 \sin(st)^2}{4s^2}. \end{aligned} \quad (20)$$

In this case the populations oscillate with a frequency of $2s$ and the amplitude of the oscillations is given by $\Omega^2/8s^2$. The time averaged population in the excited state is also given by $\bar{p}_1 = \Omega^2/8s^2 = 1/2(\Delta^2/\Omega^2 + 1)$, i.e. for $\Delta \gg \Omega$ this is approximately $\bar{p}_1 \approx \Omega^2/2\Delta^2$. This result gives the criterion for when the effect of a classical laser beam on an atomic transitions will be small and can be neglected (i.e. when the rotating wave approximation can be used).

4.2.2 Phase gate S

If in the above expression Eq. (19) we choose $s = \Delta/2$ (i.e. $\Omega = 0$) and let the system evolve for a time t with $\Delta t = \pi/2$ we find

$$\mathcal{U}(\pi/2\Delta) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = S \quad (21)$$

and thus applying no laser pulse implements a phase gate.[2]

5 Trapping atoms in light fields

So far we have assumed that the atoms are trapped at a fixed position and that the wave function is localized to a region much smaller than the wave length of the laser light (Lamb-Dicke regime). There are several ways of trapping atoms, like e.g. ion traps and magnetic traps. Here we want to briefly consider how optical trapping can be achieved. We consider an atom interacting with a classical laser light which is far detuned $\Delta \gg \Omega$. As above we adiabatically eliminate the excited state and find a resulting Hamiltonian for the ground state level (including the motional part with momentum operator $\hat{p}$) given by

$$H = \frac{\hat{p}^2}{2m} - \frac{\Omega^2}{4\Delta} \quad (22)$$

where m is the mass of the atoms and the spatial dependence of the Rabi frequency Ω is determined by the spatial dependence of the electric field. For a standing wave with wave number k it is given by $\Omega(x) = \bar{\Omega} \sin(2kx)$. In this case the resulting potential corresponds to an optical lattice which can be used to store arrays of atoms. Around the potential minima the potential is given by $2\bar{\Omega}k^2x^2$ corresponding to a harmonic oscillator potential with trapping frequency $\nu^2 = 4\bar{\Omega}k^2/m$.

[2] This is since we have chosen a frame rotating with the laser frequency ω (or detuning in the case of Raman transitions) and not the atomic transition frequency, i.e. the relative phase is measured with respect to the phase ωt and not to $(\omega_1 - \omega_0)t$.

6 Nuclear magnetic resonance (NMR)

We look at atoms/molecules in a homogeneous time dependent magnetic field $\mathbf{B}(t)$. The coupling between atom and magnetic field yields an interaction Hamiltonian $H_{\text{int}} = -\hat{\mu}_B \cdot \mathbf{B}$ where $\hat{\mu}_B$ is the magnetic dipole moment operator for the atom. Like above, in the case of a laser coupling to an atom, we assume that by choosing the frequency ω of the magnetic field and the states of the atom appropriately we reduce the dynamics of the system to a two level atom. In particular we use a magnetic field of the form $\mathbf{B} = \{B_\perp \cos(\omega t), B_\perp \sin(\omega t), B_0\}$. This magnetic field leads to an interaction Hamiltonian for the two states under consideration in the rotating frame H_{red} where the detuning is given by $\Delta = \omega_0 - \omega$ with ω_0 the splitting of the two levels $|0\rangle$ and $|1\rangle$ in the magnetic field B_0. The Rabi frequency Ω is proportional to the magnetic field $B_\perp$.

- The atomic states involved can both be in the ground state manifold and are therefore metastable. No excited state is involved in the transition between the two states and no optical spontaneous emission will take place.
- These Rabi transitions can be performed with a spatially homogeneous magnetic field and no trapping of the atoms is necessary.
- A large number of atoms/molecules can be used at the same time to enhance the signal (magnetization) of the system
- Manipulating atomic hyperfine states with magnetic fields is technically easier than quantum optical setups using lasers.
- If the atoms are not trapped there is less control and it is not yet known how NMR systems can be scaled to large numbers of qubits.

6.1 Relaxation times

6.1.1 Spin-lattice relaxation time T_1

The time T_1 is the time constant at which the energy of the state $|1\rangle$ (whose energy is shifted by the magnetic field) relaxes. In solid state systems this relaxation is due to interactions between the spin and the lattice of the solid body (thus the name).

6.1.2 Spin-Spin relaxation time T_2

This time T_2 is the time at which the phase of the spin (i.e. the relative phase in a superposition of $|0\rangle$ and $|1\rangle$ dephases or relaxes. This relaxation is often due to spin-spin interaction. In solid state systems we usually have $T_2 \ll T_1$ and in fluids $T_2 \approx T_1$.

6.2 Spin echo

In a perfectly homogeneous magnetic field all spins of an ensembles of atoms oscillate at the same frequency. If the magnetic field is not perfectly homogeneous then the spins oscillate at different frequencies and dephase. The effect of this dephasing can be avoided by the so called spin echo method. Without applied Rabi frequency $\Omega \propto B_\perp = 0$ the state $|1\rangle$ of atom j rotates at a frequency ω_{0j} which is proportional to the local magnetic field at the position at this atom. Since they are all slightly different from each other the resulting magnetization will dephase.

If a π pulse is applied after a certain time τ the dephasing can be undone. Let us consider a special example for the initial state $|\psi\rangle$

$$|\psi(0)\rangle_j = |0\rangle_j + e^{i\phi} |1\rangle_j \tag{23}$$

in each atom. This state evolves according to

$$|\psi(t)\rangle_j = |0\rangle_j + e^{i\phi - i\omega_{0j}t} |1\rangle_j \tag{24}$$

and dephases due to different ω_{0j}. If a π pulse is applied after time $t = \tau$ the state becomes

$$|\psi(t)\rangle_j = |1\rangle_j + e^{i\phi - i\omega_{0j}\tau} |0\rangle_j \tag{25}$$

and the subsequent evolution yields a state at time $t = 2\tau$

$$|\psi(t)\rangle_j = e^{-i\omega_{0j}\tau} |1\rangle_j + e^{i\phi - i\omega_{0j}\tau} |0\rangle_j \tag{26}$$

which, after another π pulse, up to a global phase, is identical to the initial state.

7 Building a quantum computer

Up to now we have only considered the basic building blocks of a quantum computer, the qubits, and investigated how to perform quantum operations on them. When building a quantum computer these are the first important steps. However, to successfully build a quantum computer one also has to implement the following:

- Quantum operations which entangle two qubits: e.g. a CNOT gate. A CNOT gate has the following truth table

$$\begin{aligned} |0\rangle |0\rangle &\rightarrow |0\rangle |0\rangle \\ |0\rangle |1\rangle &\rightarrow |0\rangle |1\rangle \\ |1\rangle |0\rangle &\rightarrow |1\rangle |1\rangle \\ |1\rangle |1\rangle &\rightarrow |1\rangle |0\rangle \end{aligned} \tag{27}$$

  i.e. the second qubit is inverted if the first one is in state $|1\rangle$ and otherwise remains unchanged. Quantum gates like this require some kind of controlled interactions between the two qubits. For ions one can use the Coulomb interaction, for neutral atoms s-wave interactions or dipole-dipole interactions between the atoms. In NMR spin-spin interactions within one molecule may be employed for implementing two qubit gates. Any unitary time evolution on a set of qubits can be decomposed into a set of single-qubit gates and the CNOT gate. In addition the decoherence time of the system has to be much larger than any of the gate times.

- State preparation: i.e. a method to initialize the state of the qubits to a certain state usually $|0\rangle$.

- State readout: After the computation is performed one has to be able to read out the result. This corresponds to performing measurements on single qubits of the quantum register.

- Scalability: The system performing quantum computations should be scalable, i.e. it should be known how to increase the number of qubits once gate operations between a small number of qubits are possible.
- Transmission of quantum information: In addition to the above criteria one also has to devise a scheme that allows to reliably transfer quantum information from a steady qubit (e.g. an atom) to flying qubits as for instance photons which may store quantum information in their polarization and can be transmitted faithfully.

A Problems

1. Calculate the dipole matrix elements for the hydrogen atom μ_{kl} for circular and linearly polarized light, where k, l labels the states of the atom. Find the selection rules for dipole allowed transitions.
2. For a ^{87}Rb atom estimate the required Rabi frequency for creating an optical lattice with a ground state size smaller than 15% of the laser wave length and a probability of 1% of spontaneously emitting a photon within a trapping time of 1s.
3. Consider the energy level structure of the hydrogen like atom ^{87}Rb:
   - Choose two hyperfine levels of the ground state manifold that can be used by represent a qubit. Which laser polarizations could be used to implement single qubit gates via Raman transitions? Assuming a maximum optical Rabi frequency of $\Omega_0 = \Omega_1 =$ 100MHz find the gate time for a π pulse if the error probability due to spontaneous emission is to be smaller than 1%.
   - Now consider the same ground state manifold and a qubit where gate operations can be done via magnetic fields. Choose magnetic field parameters with $B_0 \gg B_\perp$ and a maximum magnetic field of $B_0 = 3T$, $B_\perp = B_0/100$. How fast can a π pulse be performed in this case?

^{87}Rb: Hydrogen like atom with ground state manifold $5S_{1/2}$, nuclear spin $I = 3/2$, mass $m = 1.44 \times 10^{-25}$kg. The ground state hyperfine splitting is $\omega_{\mathrm{HF}} = 6.8$GHz between total spin states $F = 2$ and $F = 1$. Transition lines: D1 line $\lambda_1 = 794.8$nm ($5S_{1/2} \leftrightarrow 5P_{1/2}$) and D2 line $\lambda_2 = 780$nm ($5S_{1/2} \leftrightarrow 5P_{3/2}$). Life time of the $5P$ states due to spontaneous emission $\tau \approx 27$ns.