Revision class TT: Quantum Information Processing

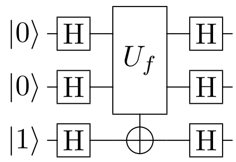
DJ and JJ

C2 2011 Question 5: Deutsch–Jozsa

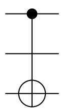
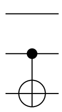
This algorithm is only touched on in the lectures, but the students should be able to work out the details on the fly.

Oracles [7 marks]

Question: The Deutsch–Jozsa algorithm permits the efficient identification of classical binary functions from n bits to 1 bit which are either constant or balanced, and can be implemented in the case $n = 2$ using the network below

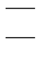
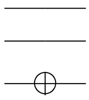


where the last qubit is an ancilla, and the networks below act as oracle implementations of two of the six balanced functions.

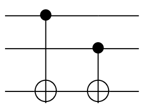
$f_{0011} =$

$f_{0101} =$


Draw labelled networks for oracle implementations of the four remaining balanced functions, the two constant functions, and the unbalanced function f_{0001}.

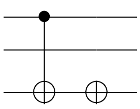
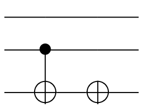
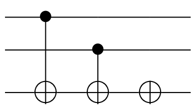
Solution: The constant functions are trivial

$f_{0000} =$

$f_{1111} =$


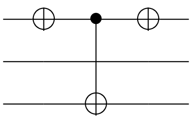
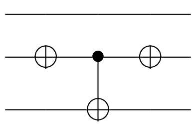
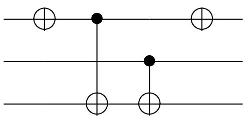
and the third balanced function is

$f_{0110} =$


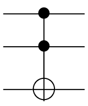
The final three balanced functions can be obtained *either* by negating the ancilla in addition to the control gates (say, afterwards)

$f_{1100} =$

$f_{1010} =$

$f_{1001} =$


or, a bit more messily, by negating the control lines before and after the gates

$f_{1100} =$  $f_{1010} =$  $f_{1001} =$ 

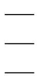
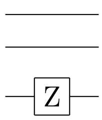
Note that for f_{1001} you must NOT either one of the control lines but not both! The network for the unbalanced function is

$f_{0001} =$ 

Algorithm [13 marks]

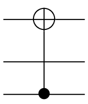
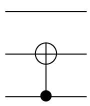
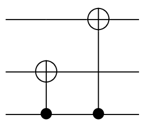
Question: Find the final state of the three qubits for the two constant functions and the six balanced functions using either matrix methods (you may find it useful to factor out the ancilla qubit and any global phases) or circuit identities, and determine the probability of finding both input qubits in the final state $|0\rangle$ in each case. Repeat this calculation for the function f_{0001} and discuss whether the Deutsch–Jozsa algorithm can be useful with unbalanced functions.

Solution: The constant functions are best tackled using the circuit identities $\mathrm{H}^2 = \mathbf{1}$ and $\mathrm{HXH} = \mathrm{Z}$ to simplify them to

f_{0000}: $|001\rangle$  $|001\rangle$ f_{1111}: $|001\rangle$  $-|001\rangle$

and so $P(0,0) = 1$ for constant functions.

The balanced functions are only slightly more complex to tackle in the same way, using the fact that the effect of applying Hadamard gates to both qubits before and after a CNOT gate is simply to reverse the roles of control and target qubits. Thus

f_{0011}: $|001\rangle$  $|101\rangle$ f_{0101}: $|001\rangle$  $|011\rangle$ f_{0110}: $|001\rangle$  $|111\rangle$

as the ancilla bit starts in $|1\rangle$, and so the CNOT gates can be treated as simple NOTs. The remaining three balanced functions simply involve extra NOT gates on the ancilla, which turn into extra Z gates on the ancilla, and so the results are just

$$f_{1100}: \quad -|101\rangle \qquad f_{1010}: \quad -|011\rangle \qquad f_{1001}: \quad -|111\rangle$$

and so $P(0,0) = 0$ for balanced functions. If you use the oracle form with NOT gates on the control lines then the calculations are a bit more complicated: start by showing that HX=ZH and XH=HZ, and so the circuits simplify as before, except that there will be a pair of Z gates surrounding the target of each CNOT gate; as the first Z gate is applied to $|0\rangle$ and the second to $|1\rangle$ the overall result will be negation of the final state as before.

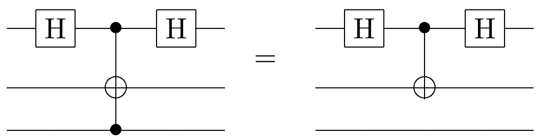
Alternatively you could tackle the balanced functions using brute force matrix methods. Start off by proving that the effect of using an ancilla qubit in state $|1\rangle$ with a pair of Hadamard gates surrounding the CNOT gates is to negate the ancilla qubit, and this phase shift kicks back onto the control qubits . Then we can factor out the ancilla qubit and replace the classical oracles by quantum oracles which are diagonal matrices with entries of 1 where the function gives 0 and -1 where the function gives 1; thus $U_{0011} = \{1, 1, -1, -1\}$ and so on . Brute force calculations then gives the previous states and probabilities . The constant functions could in principle be done in the same way.

The unbalanced function can be tackled using either circuit identities or brute force multiplication. For f_{0001} the brute force calculation is

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \xrightarrow{\mathrm{H}\otimes\mathrm{H}} \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \xrightarrow{U} \frac{1}{2}\begin{pmatrix}1\\1\\1\\-1\end{pmatrix} \xrightarrow{\mathrm{H}\otimes\mathrm{H}} \frac{1}{2}\begin{pmatrix}1\\1\\1\\-1\end{pmatrix}$$

and so $P(0,0) = 1/4$ for this case (and indeed any case). Thus unbalanced functions can look like either balanced or constant functions, and no useful information can be obtained. The circuit identity approach is also fine as long as you hold your nerve, giving something like

f_{0001}:



(the second stage relying on the ancilla being in state $|1\rangle$, so the Toffoli gate reduces to a CNOT). At this stage it is easiest to go to brute force on the two upper qubits.

Trapped ions [5 marks]

Question: Describe briefly how to implement Hadamard and CNOT gates in a trapped ion quantum computer.

Solution: Hadamard gates can be implemented using a laser beam focussed on the ion being addressed, controlling the power, length and phase to get an appropriate rotation. The Hadamard gate is not a rotation in the xy plane but can be constructed out of such gates. (Alternatively you can use off-resonance excitation to implement the Hadamard directly).

CNOT gates can be implemented by first using a laser pulse to selectively excite the vibrational mode of the control ion conditional on its internal state. As the ions are all coupled by a the Coulomb force they share common vibrational states the target ion will also have its vibrational state selectively excited. Then use a laser pulse on the target ion to modify its internal state conditional on its vibrational state. (Don't need to add details or talk about de-exciting the vibrational mode; the three key points above will do.)

C2 2011 Question 6: Decoherence and error correction

In some cases there are several obvious methods of solution.

Basics [5 marks]

Question: Write down the matrix form of the quantum gate ϕ_z and find the effect of applying this to a general state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Use explicit matrix methods to show that X=HZH, and hence or otherwise find the effect of applying ϕ_x to $|\psi\rangle$.

Solution: Starts with some basic standard calculations:

$$\phi_z = \exp(-i\phi\sigma_z/2) = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}$$

$$\phi_z|\psi\rangle = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha e^{-i\phi/2} \\ \beta e^{i\phi/2} \end{pmatrix}$$

$$\mathrm{HZH} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

For next bit the *hence* approach relies on spotting that the above implies that $\phi_x = \mathrm{H}\phi_z\mathrm{H}$ and then using

$$\phi_x|\psi\rangle = \mathrm{H}\phi_z\mathrm{H}|\psi\rangle = \mathrm{H}\phi_z\frac{1}{\sqrt{2}}\begin{pmatrix} \alpha+\beta \\ \alpha-\beta \end{pmatrix} = \frac{1}{\sqrt{2}}\mathrm{H}\begin{pmatrix} (\alpha+\beta)e^{-i\phi/2} \\ (\alpha-\beta)e^{i\phi/2} \end{pmatrix}$$

and so on; it is probably more sensible to do this *otherwise* using

$$\phi_x = \exp(-i\phi\sigma_x/2) = \cos(\phi/2)\mathbf{1} - i\sin(\phi/2)\sigma_x = \begin{pmatrix} \cos(\phi/2) & -i\sin(\phi/2) \\ -i\sin(\phi/2) & \cos(\phi/2) \end{pmatrix}$$

$$\phi_x|\psi\rangle = \begin{pmatrix} \cos(\phi/2)\alpha - i\sin(\phi/2)\beta \\ \cos(\phi/2)\beta - i\sin(\phi/2)\alpha \end{pmatrix}$$

Decoherence [11 marks]

Question: Consider an ensemble of qubits which start in the state $|\psi\rangle$ and then experience either a ϕ_z gate, an identity gate, or a ϕ_{-z} gate, chosen independently at random for each qubit in the ensemble. Show that the final state is identical to that of an ensemble of qubits which either experience a Z gate with some probability p, or are left untouched with probability $1-p$, and find the relationship between p and ϕ.

Consider the special case where $|\psi\rangle$ lies on the equator of the Bloch sphere. Calculate the purity of the final state, and find the value of ϕ which reduces the purity of the state to the minimum possible value. Use the Bloch sphere picture to explain why this occurs.

Solution: We first need to calculate the density matrix

$$\rho = \frac{1}{3}\left(\phi_z|\psi\rangle\langle\psi|\phi_z^\dagger + |\psi\rangle\langle\psi| + \phi_{-z}|\psi\rangle\langle\psi|\phi_{-z}^\dagger\right)$$

The first term is given by

$$\phi_z|\psi\rangle\langle\psi|\phi_z^\dagger = \begin{pmatrix} \alpha e^{-i\phi/2} \\ \beta e^{i\phi/2} \end{pmatrix}\begin{pmatrix} \alpha^* e^{i\phi/2} & \beta^* e^{-i\phi/2} \end{pmatrix} = \begin{pmatrix} \alpha\alpha^* & \alpha\beta^* e^{-i\phi} \\ \beta\alpha^* e^{i\phi} & \beta\beta^* \end{pmatrix}$$

and the second and third terms are equivalent with ϕ replaced by 0 and $-\phi$ respectively, so

$$\rho = \begin{pmatrix} \alpha\alpha^* & \alpha\beta^*(1+2\cos\phi)/3 \\ \beta\alpha^*(1+2\cos\phi)/3 & \beta\beta^* \end{pmatrix}$$

For the second definition we use

$$\mathrm{Z}\,|\psi\rangle\,\langle\psi|\,\mathrm{Z} = \begin{pmatrix} \alpha\alpha^* & -\alpha\beta^* \\ -\beta\alpha^* & \beta\beta^* \end{pmatrix}$$

so

$$\rho' = (1-p)\times|\psi\rangle\,\langle\psi| + p\times\mathrm{Z}\,|\psi\rangle\,\langle\psi|\,\mathrm{Z} = \begin{pmatrix} \alpha\alpha^* & \alpha\beta^*(1-2p) \\ \beta\alpha^*(1-2p) & \beta\beta^* \end{pmatrix}$$

and $\rho' = \rho$ iff $(1+2\cos\phi)/3 = 1-2p$, that is $p = (1-\cos\phi)/3$ or $\phi = \arccos(1-3p)$.

States on the equator have $\alpha = 1/\sqrt{2}$ and $\beta = e^{i\gamma}/\sqrt{2}$ so the density matrix simplifies to

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\gamma}(1-2p) \\ e^{i\gamma}(1-2p) & 1 \end{pmatrix}.$$

For purity calculation don't need to do a full matrix multiplication as only need the sum of two terms giving

$$y = (1/2)^2\left[1^2 + (1-2p)^2 + (1-2p)^2 + 1^2\right] = \frac{1}{2}\left[1 + (1-2p)^2\right]$$

The minimum purity occurs at $dy/dp = 0$ which is at $p = 1/2$, and so $\phi = \arccos(1-3/2) = 2\pi/3 = 120°$. Thus the random process produces three states on the equator all equally spaced by 120°, and these sum to the maximally mixed state at the centre of the sphere, which has the lowest possible purity.

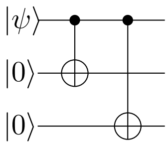
Error correction [9 marks]

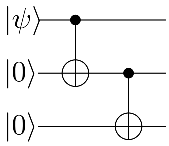
Question: Describe an error-correction network which can correct spin-flip errors using three physical qubits to encode one logical qubit. Explain why this network will also correct random ϕ_x gates, and discuss how the effectiveness varies with ϕ. How could this network be modified to correct random ϕ_z gates instead?

Solution: The two code words used are $|0_L\rangle$=$|000\rangle$ and $|1_L\rangle$=$|111\rangle$, and an arbitrary superposition state is encoded as

$$|\psi_L\rangle = \alpha\,|0_L\rangle + \beta\,|1_L\rangle = \alpha\,|000\rangle + \beta\,|111\rangle$$

which can be achieved using either of the encoding networks shown below.

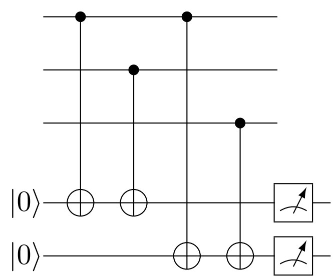




After the error process (assume that at most one of the three physical qubits has been flipped) this state is converted to

$$|\psi_L\rangle \longrightarrow \begin{cases} \alpha\,|000\rangle + \beta\,|111\rangle & \text{no error} \\ \alpha\,|100\rangle + \beta\,|011\rangle & \text{bit 1 flipped} \\ \alpha\,|010\rangle + \beta\,|101\rangle & \text{bit 2 flipped} \\ \alpha\,|001\rangle + \beta\,|110\rangle & \text{bit 3 flipped} \end{cases}$$

depending on the exact form of the error. The task is to identify the error while learning nothing about α and β. This can be achieved using the following network, which requires two additional (ancilla) qubits



where the bottom two qubits are the ancillas. If no error has occurred then both ancillas will end up in state $|0\rangle$, while if an error has occurred then either or both of the ancillas will end up in state $|1\rangle$. The first ancilla can only end up in state $|1\rangle$ if the physical qubits 1 and 2 have different values, while the second ancilla can only end up in state $|1\rangle$ if the physical qubits 1 and 3 have different values. Thus ancilla results of 00 indicates no error, 10 bit 2 error, 01 bit 3 error, and 11 bit 1 error. By this means the error can be detected and corrected. Finally the encoded state can be decoded by reversing the encoding network.

This network will also correct ϕ_x errors, as these are equivalent to probabilistic X errors, but will only be effective if ϕ is small so that the corresponding error probability is small. Finally ϕ_x gates are the same as random ϕ_z gates if Hadamard gates are applied to all *physical* qubits before and after the error process.

C2 2011 Question 7: Atom dynamics and Ramsey fringes

Good qubits [5 marks]

Question: Not all internal states of an atom are suitable for representing basis states of a qubit. State two properties of states that make a 'good' qubit. Two hyperfine states of an atom $|g\rangle$ and $|e\rangle$ with energy difference $\hbar\omega$ are used to represent a qubit. Briefly explain how coherent Rabi-oscillations can be driven between these two states. What is the effect of a resonant $\pi/2$ pulse with phase zero on the states $|g\rangle$ and $|e\rangle$?

Solution: Both qubit states should be long-lived compared to typical gate operation times and they must be initializable (other properties arising from the diVincenzo criteria like being measurable and being able to implement gates are also ok). Transitions between hyperfine states can be driven using Raman transitions via a detuned optically excited state (or directly using microwaves, both answers are ok). Rabi flopping realizes the Hamiltonian $H = \Omega\sigma_x/2$ and a pulse of duration T with $\Omega T = \pi/2$ thus drives $|g\rangle \to |g\rangle - \mathrm{i}\,|e\rangle$ and $|e\rangle \to |e\rangle - \mathrm{i}\,|g\rangle$.

Atom decay [10 marks]

Question: An atom is initially, at time $t = 0$, prepared in the state $|e\rangle\langle e|$, and then decays to the state $|g\rangle\langle g|$ at a rate γ. Calculate the density matrix of the atom and its entropy as a function of time and give a physical reason why this evolution cannot be unitary. What is the time evolution if the atom starts in $|g\rangle\langle g|$? The time evolution of the initial operator $|e\rangle\langle g|$ is given by

$$|e\rangle\langle g| \to \mathrm{e}^{-\mathrm{i}\omega t-\gamma t/2}\,|e\rangle\langle g| \ .$$

What is the evolution of the operator $|g\rangle\langle e|$?

Describe the trajectory followed by the initial density matrix $|e\rangle\langle e|$ on the Bloch sphere where the north pole corresponds to $|g\rangle\langle g|$ and the south pole to $|e\rangle\langle e|$ qualitatively as a function of t. Compare this to the trajectory of the state of an atom driven by a π pulse from $|e\rangle\langle e|$ to $|g\rangle\langle g|$. How does the entropy change along these trajectories?

Solution: The initial state evolves according to $|e\rangle\langle e| \to p_{ee}(t)|e\rangle\langle e| + p_{gg}(t)|g\rangle\langle g|$ with $p_{ee} + p_{gg} = 1$. The atom decays at rate γ meaning that $\dot{p}_{ee} = -\gamma p_{ee}$ which yields $p_{ee} = \mathrm{e}^{-\gamma t}$ and $p_{gg} = 1 - \mathrm{e}^{-\gamma t}$ satisfying the initial condition. The entropy is given by $S = -p_{ee}\log_2 p_{ee} - p_{gg}\log_2 p_{gg}$ which changes with time and thus the evolution cannot be unitary. There is no decay from the ground state so that the initial state $|g\rangle\langle g| \to |g\rangle\langle g|$ remains unchanged. The operator $|g\rangle\langle e|$ is the adjoint of the one given in the question and so evolves according to $|g\rangle\langle e| \to \mathrm{e}^{\mathrm{i}\omega t - \gamma t/2}|g\rangle\langle e|$.

The trajectory followed by the initial state $|e\rangle\langle e|$ is a straight line from the south-pole to the north-pole of the Bloch sphere passing through its centre which is the maximally mixed state. The entropy increases from 0 to 1 while the trajectory moves from the south-pole to the centre of the Bloch sphere and then decreases until it reaches zero at the north pole. If instead the initial state is driven by a Rabi pulse it follows a great circle on the surface of the sphere from the south-pole to the north-pole, the state remains pure and the entropy is always zero.

Ramsey fringes [10 marks]

Question: The atom now starts in $|g\rangle\langle g|$ and an instantaneous $\pi/2$ pulse is applied at time $t = 0$. It is then allowed to evolve for a time τ before a second instantaneous $\pi/2$ pulse is applied. The atom is measured immediately afterwards. Calculate the probability of finding the atom in state $|e\rangle$ in this measurement as a function of τ for a given ω and the cases where $\gamma = 0$ and where $\gamma \neq 0$. Briefly discuss the implications of this result for using atomic states $|g\rangle$ and $|e\rangle$ as a qubit or as the two paths of an atom interferometer.

Solution: After the $\pi/2$ pulse the atom is in state $(|g\rangle\langle g| + \mathrm{i}|g\rangle\langle e| - \mathrm{i}|e\rangle\langle g| + |e\rangle\langle e|)/2$. During time τ this state evolves into $((2-p_{ee})|g\rangle\langle g| + \mathrm{i}\mathrm{e}^{\mathrm{i}\omega\tau - \gamma\tau/2}|g\rangle\langle e| - \mathrm{i}\mathrm{e}^{-\mathrm{i}\omega\tau - \gamma\tau/2}|e\rangle\langle g| + p_{ee}|e\rangle\langle e|)/2$. The second $\pi/2$ pulse is then applied and we only work out the contributions to the term $|e\rangle\langle e|$ which determine the probability p of finding the atom in state $|e\rangle$. These are given by $p = (2 + \mathrm{e}^{\mathrm{i}\omega\tau - \gamma\tau/2} + \mathrm{e}^{-\mathrm{i}\omega\tau - \gamma\tau/2})/4$ which simplifies to $p = (1 + \mathrm{e}^{-\gamma\tau/2}\cos(\omega\tau))/2$. For $\gamma = 0$ this gives standard Ramsey fringes oscillating around $1/2$ with frequency ω. When $\gamma \neq 0$ the fringes damp out with increasing τ and vanish for $\tau \gtrsim 1/\gamma$ indicating the loss of coherence. Interferometry experiments and quantum computations thus need to be carried out in times $t \lesssim 1/\gamma$ if effects of quantum coherence are to be exploited.

C2 2011 Question 8: Procrustean method

Introduction [5 marks]

Question: The computational basis states of a qubit are encoded as as horizontal $|H\rangle$ and vertical $|V\rangle$ polarizations of a photon. Which polarizations do the X and the Y basis states correspond to in this encoding? Draw schematic experimental setups using polarizing beam splitters (PBS), photo-detectors and wave-plates for measuring the qubit in each of the three bases.

Solution: The eigenstates in the X basis are $\propto |H\rangle \pm |V\rangle$ which are $\pm\pi/4$ polarized photons. In the Y basis the states are $\propto |H\rangle \pm \mathrm{i}|V\rangle$ which are right/left circularly polarized photons. A polarizing beam splitter (PBS) followed by a detector in each arm will measure in the Z basis. For the X basis the PBS can be rotated by $\pi/4$ or alternatively a $\lambda/2$ wave plate at an angle $\pi/8$ can be placed into the photon path to rotate the X basis into the Z basis and then measure in the Z basis. For measuring in the Y basis a $\lambda/4$ plate at angle $\pi/4$ is required before the PBS.

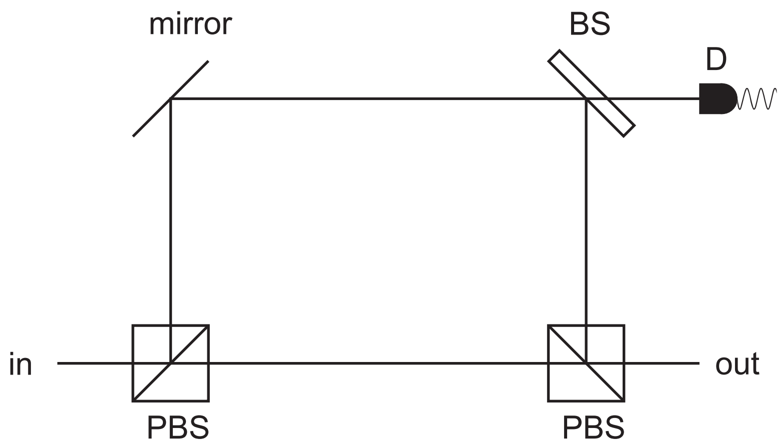
Photon source [8 marks]

Question: A two photon source produces polarization entangled photons in the state $|\alpha\rangle = \sqrt{\alpha}\,|VV\rangle + \sqrt{1-\alpha}\,|HH\rangle$ with $0 \leq \alpha \leq 1$. Calculate the joint entropy, the entropy of the reduced density matrix of each photon, and the mutual information between the photons as a function of α. Discuss the significance of the mutual information for measurements in the cases where $|\alpha\rangle$ is a product state and where it is a maximally entangled state.

Solution: The two-photon state is pure and thus the joint entropy is $S = 0$. The reduced density matrix is $\rho = \alpha\,|V\rangle\langle V| + (1-\alpha)\,|H\rangle\langle H|$, the same for each photon, with an entropy of $S_p = -\alpha\log_2\alpha + (\alpha - 1)\log_2(1-\alpha)$. The mutual information is thus $S(1:2) = 2S_p$. The state $|\alpha\rangle$ is a product state for $\alpha = 0, 1$ where $S(1:2) = 0$ and the reduced density matrices are pure. Since $S(1:2) = 0$ measuring one photon will not provide any information about the state of the other photon. The state $|\alpha\rangle$ is maximally entangled for $\alpha = 1/2$ where $S(1:2) = 2$ and the reduced density matrix of each photon is maximally mixed. Since $S(1:2) = 2$ measuring one photon will project the other photon into a pure state known to the observer. The mutual information exceeding the classical limit of 1 indicates entanglement between the two photons thus allowing stronger than classical correlations between measurement results in different bases.

Procrustean methods [12 marks]

Question: Alice and Bob receive one photon of $|\alpha\rangle$, respectively. Bob sends his photon through



the device shown in the figure where the PBSs transmit photons in $|V\rangle$ and reflect those in state $|H\rangle$; the beam splitter (BS) has reflectivity R for both polarizations. Alice and Bob only keep those photon pairs where the detector D does not click. Work out the state of these remaining photons. How must R be chosen so that these photon pairs are maximally entangled? For which values of α is such a choice of R physically possible? If the photon source produces N pairs per second how many maximally entangled photon pairs do Alice and Bob obtain per second?

Solution: A $|V\rangle$ photon goes straight through the device, i.e. $|V\rangle_{\rm in} \to |V\rangle_{\rm out}$. For an $|H\rangle$ photon the device simplifies to a BS either transmitting the photon towards the detector D or reflecting it to the output port, i.e. $|H\rangle_{\rm in} \to \sqrt{R}\,|H\rangle_{\rm out} + \sqrt{1-R}\,|D\rangle$, where $|D\rangle$ indicates Bob's photon ending up in the detector. The overall state thus turns into $|\alpha\rangle \to \sqrt{\alpha}\,|VV\rangle + \sqrt{R - R\alpha}\,|HH\rangle + \sqrt{(1-R)(1-\alpha)}\,|HD\rangle$. Only the first two parts of the state are kept and this is a maximally entangled state for $\alpha = R(1-\alpha)$ leading to $R = \alpha/(1-\alpha)$. R must fulfill $0 \leq R \leq 1$ so that this implementation only works for $0 \leq \alpha \leq 1/2$. The probability of no detector click is 2α so that Alice and Bob obtain $2\alpha N = 2RN/(1+R)$ photons per second.