Two Quantum Optics Questions - 2012

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1. A neutron beam is polarized parallel to a uniform magnetic field B. The beam is then split into two halves. One half continues through a uniform magnetic field, while the other half passes though a field of the same fixed magnitude but which gradually changes its direction. The two beams are recombined and the intensity measured. The path lengths are equal so that the interference would be constructive if B were uniform. Assume that the neutrons in the first beam experience the magnetic field given by

$$B = B_0(\sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}) \ ,$$

while, in the second beam, they experience the time-dependent magnetic field of the form:

$$B(t) = B_0(\sin\theta\cos\phi(t)\hat{\mathbf{x}} + \sin\theta\sin\phi(t)\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}})$$

where the angle θ varies adiabatically from zero to 2π.

Calculate the ground state wavefunction, in the basis of σ_z, for a constant field pointing in an arbitrary direction given by angles θ and ϕ.

Calculate the relative phase between the two beams after they have passed through their respective magnetic fields, and hence the intensity of the recombined beam as a function of θ.

2. Show that the action of the squeezing operator $S(\alpha) = \exp\{\frac{1}{2}(\alpha^* a^2 - \alpha(a^\dagger)^2)\}$ on the creation and annihilation operators is given by

$$S^\dagger a S = \cosh r a - e^{i\theta}\sinh r a^\dagger \quad (1)$$
$$S^\dagger a^\dagger S = \cosh r a^\dagger - e^{-i\theta}\sinh r a \ . \quad (2)$$

You may use the fact that

$$e^{-xA}Be^{+xA} = B - \frac{x}{1!}[A,B] + \frac{x^2}{2!}[A,[A,B]] - ...$$

Hence calculate the dispersion of the quadratures $x = (a + a^\dagger)/\sqrt{2}$ and $p = (a - a^\dagger)/i\sqrt{2})$ of the electric field in the squeezed vacuum state $S(\alpha)|0\rangle$ where $\alpha = re^{i\theta}$.

Show that the minimum value of the x dispersion occurs for $\theta = 0$ and that for this particular angle, the product of dispersions satisfies the minimum uncertainty relation $\Delta x \Delta p = 1/4$.