8. Some internal states of an atom are suitable for representing basis states of a qubit. Which properties should internal atomic states have to make a 'good' qubit? Identify two atomic states of the alkali atom ^{87}Rb (nuclear spin $I = 3/2$) which possess these properties and can be used to realize a qubit. Explain how single qubit gates can be performed in this atomic qubit. [7]

A laser setup is switched on for a time 2τ and induces the two qubit SWAP gate with truth table

$$\begin{array}{ccc} |0\rangle \otimes |0\rangle & \rightarrow & |0\rangle \otimes |0\rangle \\ |0\rangle \otimes |1\rangle & \rightarrow & |1\rangle \otimes |0\rangle \\ |1\rangle \otimes |0\rangle & \rightarrow & |0\rangle \otimes |1\rangle \\ |1\rangle \otimes |1\rangle & \rightarrow & |1\rangle \otimes |1\rangle \end{array}$$

on two adjacent atomic qubits. Write down the state resulting from the application of this gate to an arbitrary two qubit product state $|\psi\rangle \otimes |\phi\rangle$. Hence, or otherwise, discuss whether the SWAP gate in combination with single qubit gates constitute a universal set of quantum gates. [4]

By turning the lasers on for a time τ, the operation $\sqrt{\text{SWAP}}$ is realized. The states $|0\rangle \otimes |0\rangle$ and $|1\rangle \otimes |1\rangle$ are unaffected by the dynamics. Calculate a matrix representation of this $\sqrt{\text{SWAP}}$ gate in the computational basis. Apply the network

$$U = H_2 \sqrt{\text{SWAP}}\, Z_1 \sqrt{\text{SWAP}} \sqrt{Z_2}\, H_2$$

to the four computational basis states. Here Z_i denotes the Z-gate applied to the i-th qubit and H_2 is the Hadamard gate on the second qubit. Extend this network using single qubit gates to realize a CNOT gate. Discuss whether the $\sqrt{\text{SWAP}}$ gate together with single qubit gates constitute a universal set of quantum gates. [14]