6. The full Hamiltonian for a two spin NMR system has the Heisenberg form

$$\mathcal{H}_{\mathrm{H}} = \frac{\omega_1}{2}\,\sigma_z \otimes \mathbf{1} + \frac{\omega_2}{2}\,\mathbf{1} \otimes \sigma_z + \frac{\omega_{12}}{2}\,\sigma \cdot \sigma,$$

where the Heisenberg coupling is given by

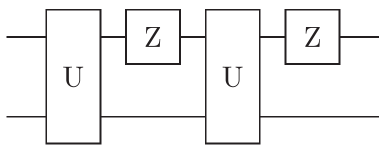
$$\sigma \cdot \sigma = \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z,$$

and factors of $\hbar$ have been dropped as usual in NMR. Write down explicit matrix forms in the computational basis for the Heisenberg coupling and for $\mathcal{H}_{\mathrm{H}}$, and use perturbation theory to determine the conditions under which $\mathcal{H}_{\mathrm{H}}$ can be approximated by the Ising form

$$\mathcal{H}_{\mathrm{I}} = \frac{\omega_1}{2}\,\sigma_z \otimes \mathbf{1} + \frac{\omega_2}{2}\,\mathbf{1} \otimes \sigma_z + \frac{\omega_{12}}{2}\,\sigma_z \otimes \sigma_z.$$

Explain why this approximation is better when larger magnetic field strengths are used. [8]

Consider the case $\omega_1 = \omega_2$ and use a rotating frame transformation to remove the Zeeman terms from the full Hamiltonian. Find an explicit matrix expression for the propagator U corresponding to evolution under the Hamiltonian in this frame for a time t and evaluate the total propagator for the network



(a modified spin-echo sequence). Explain why this network could not in fact be implemented in an NMR spin system with $\omega_1 = \omega_2$. [12]

Explain briefly the difference between quantum error correction and decoherence free subspaces. Describe a simple decoherence free subspace encoding one logical qubit in two physical spins and show that a general state of the logical qubit is protected against simultaneous Z gates. [5]