**5.*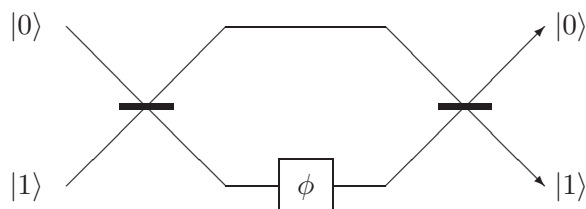* A pair of beam splitters can be used to build a Mach–Zehnder interferometer as shown in the diagram below, where the phase shifter applies $e^{i\phi}$ to photons in the lower arm $|1\rangle$ and the mirrors needed to divert the light beams have not been indicated.

Assuming that the two beam splitters can be treated as Hadamard gates and that a single photon is incident on the interferometer from the $|0\rangle$ arm on the left, calculate the probability of detecting this photon at the $|0\rangle$ and $|1\rangle$ outputs of the interferometer as a function of ϕ. [6]

The behaviour of real beam splitters can be more complex. One family of beam splitters can be described by the unitary transformation

$$U_{\mathrm{BS}} = \begin{pmatrix} e^{-i\alpha}\cos(\beta/2) & -i\sin(\beta/2) \\ -i\sin(\beta/2) & e^{i\alpha}\cos(\beta/2) \end{pmatrix}.$$

Show that the Hadamard gate is equivalent (up to a global phase) to a special case of U_{BS} by determining appropriate values of α and β, and that the general U_{BS} can be constructed from two Hadamard gates and z-rotations using the sequence

$$\alpha_z \, H \, \beta_z \, H \, \alpha_z \,,$$

where $\alpha_z = e^{-i\alpha\sigma_z/2}$ and similarly for β_z.

Suppose that an interferometer is constructed as above using two identical beam splitters of this kind with $\alpha = 0$. Calculate the probability of detecting a photon at the upper output, and show that interference fringes can still be detected. Find the values of β that give the maximum contrast to these fringes. [12]

An interferometer can also be used to perform an *interaction-free measurement* in an inefficient version of the Elitzur–Vaidman bomb testing problem. Suppose the phase shifter is replaced by a box that contains either (i) an active bomb, triggered by a perfect single-photon detector, thus implementing a projective measurement, or (ii) a passive bomb, without a detector attached. Assuming that the beam splitters can be described by Hadamard gates, describe an outcome of this experiment that allows one to deduce that a bomb is active without setting it off, and calculate the probability that this approach will be successful. [7]