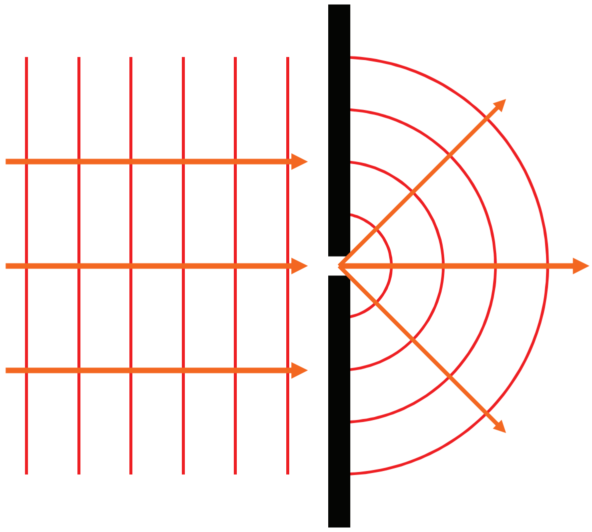
Wave Optics

- In part 1 we saw how waves can appear to move in straight lines and so can explain the world of geometrical optics
- In part 2 we explore phenomena where the wave nature is obvious not hidden
- Key words are *interference* and *diffraction*

Wave motion (1)

- See the waves lecture course for details!
- Basic form of a one-dimensional wave is $\cos(kx-\omega t-\phi)$ where $k=2\pi/\lambda$ is the *wave number*, $\omega=2\pi\nu$ is the *angular frequency*, and ϕ is the *phase*
- Various other conventions in use

Cylindrical wave



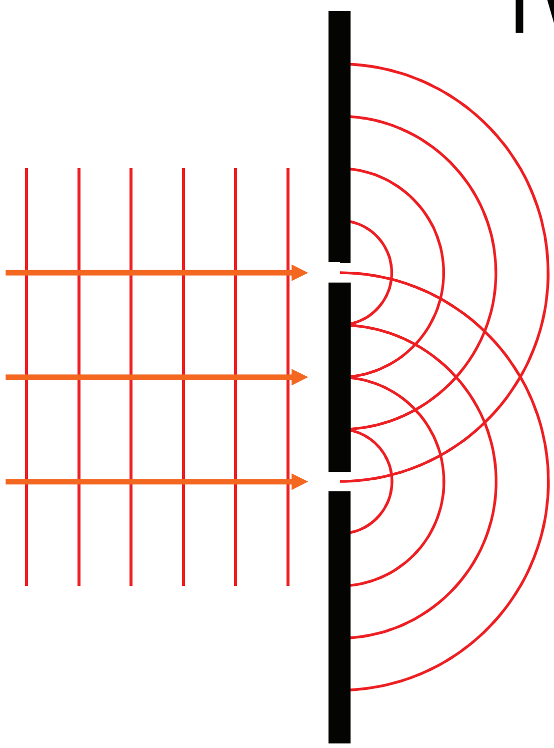
A plane wave impinging on a *infinitesimal* slit in a plate.

Acts as a point source and produces a *cylindrical wave* which spreads out in all directions.

Finite sized slits will be treated later.

Remainder of this course will use slits unless specifically stated

Two slits

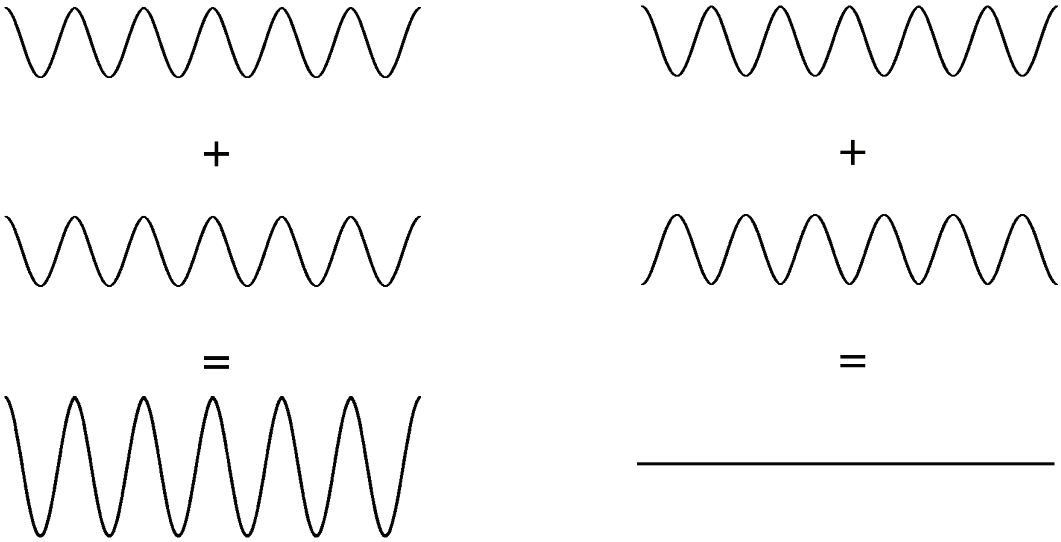


A plane wave impinging on a pair of slits in a plate will produce two circular waves

Where these overlap the waves will interfere with one another, either reinforcing or cancelling one another

Intensity observed goes as square of the total amplitude

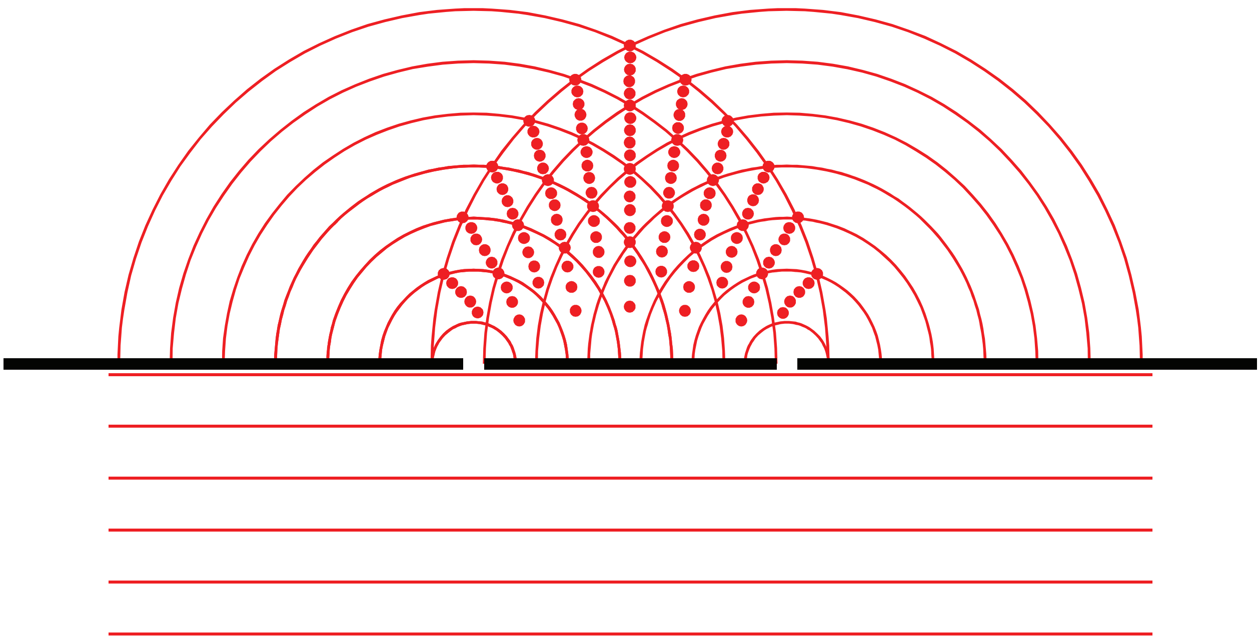
Interference



Constructive:
intensity=4

Destructive:
intensity=0

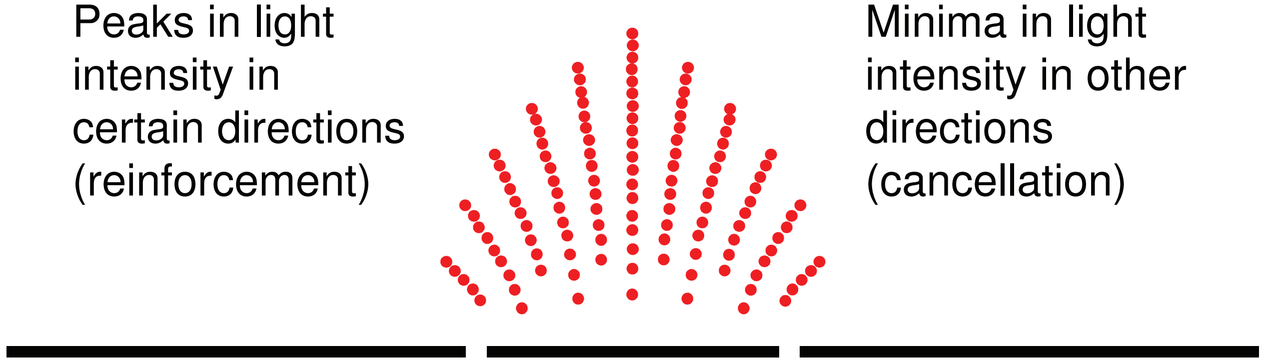
Two slit interference



Two slit interference

Peaks in light intensity in certain directions (reinforcement)

Minima in light intensity in other directions (cancellation)



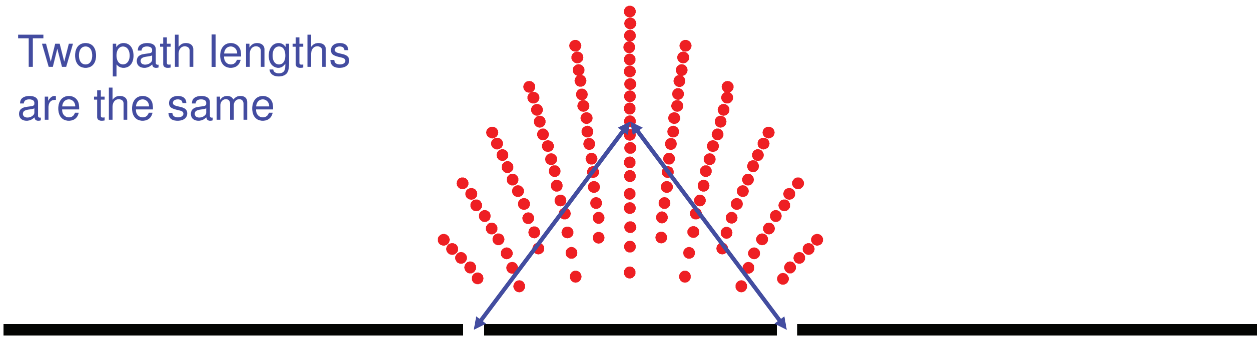
Can we calculate these directions directly without all this tedious drawing?

Two slit interference (2)

- These lines are the sets of positions at which the waves from the two slits are *in phase* with one another
- This means that the *optical path lengths* from the two slits to points on the line must differ by an integer number of wavelengths
- The amplitude at a given point will oscillate with time (not interesting so ignore it!)

Two slit interference (3)

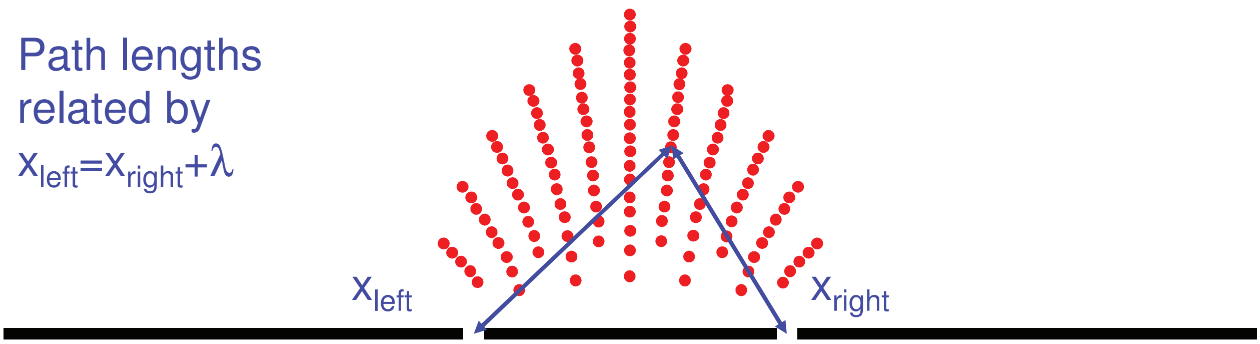
Two path lengths
are the same



Central line is locus of points at same distance from the two slits

Two slit interference (4)

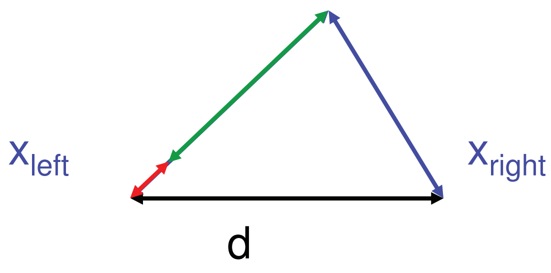
Path lengths
related by
$x_{left} = x_{right} + \lambda$



Next line is locus of points where distance from the two slits differs by one wavelength

Two slit interference (5)

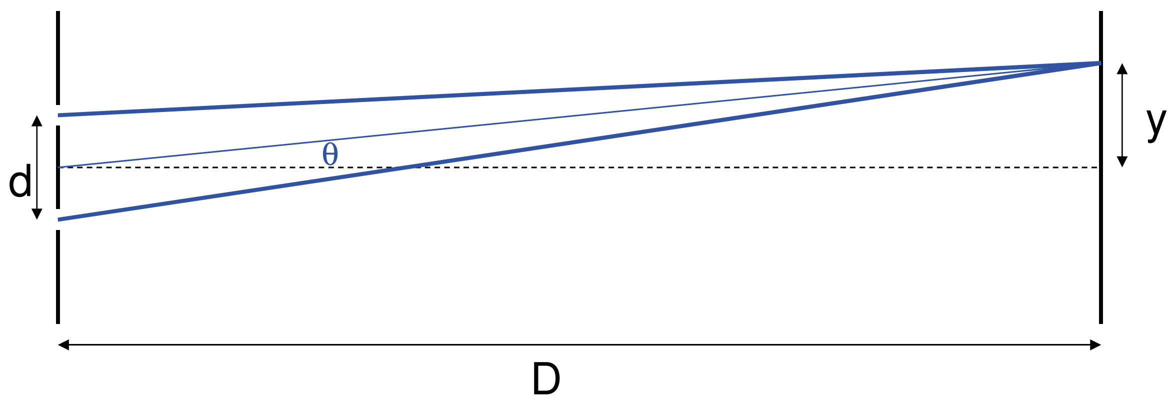
Path lengths related by $x_{left}=x_{right}+\lambda$



Next line is locus of points where distance from the two slits differs by one wavelength

Two slit interference (6)

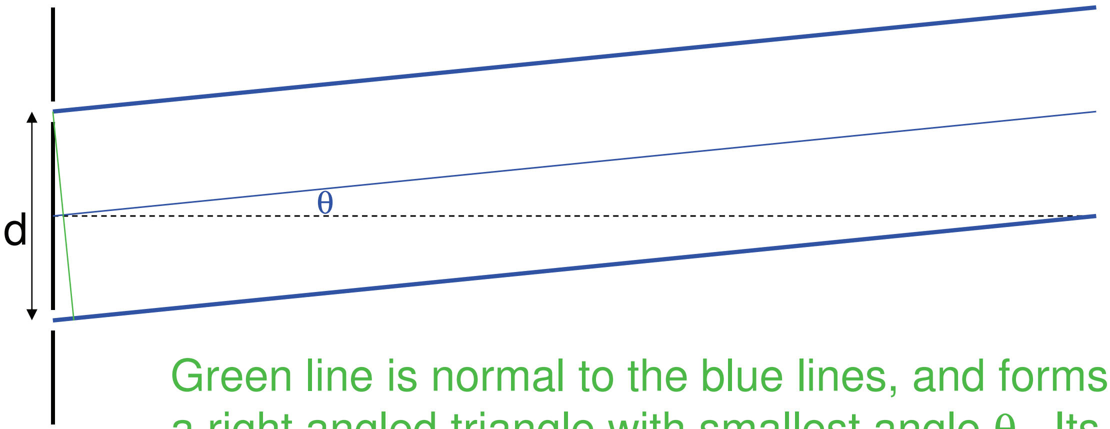
Interference pattern observed on a distant screen



As $y \ll D$ the three blue lines are effectively parallel and all make an angle $\theta \approx y/D$ to the normal. The bottom line is slightly longer than the top.

Two slit interference (7)

Close up



Green line is normal to the blue lines, and forms a right angled triangle with smallest angle θ. Its shortest side is the extra path length, and is of size $d \times \sin(\theta) \approx dy/D$

Two slit interference (8)

View at the screen

Central fringe midway between slits

Bright fringes seen when the extra path length is an integer number of wavelengths, so $y = n\lambda D/d$

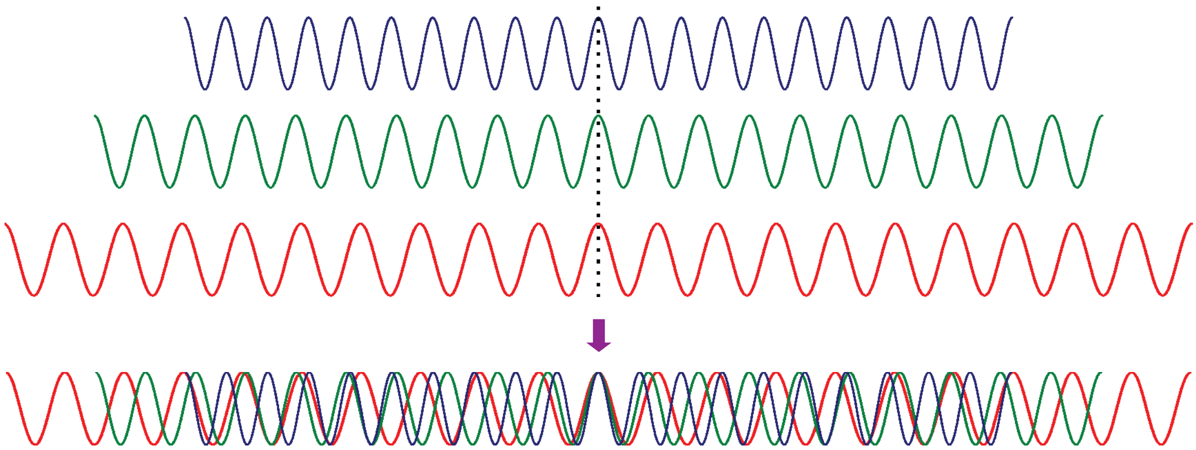
Dark fringes seen when $y = (n+½)\lambda D/d$

Taking λ=500nm, d=1mm, D=1m, gives a fringe separation of 0.5mm

Practicalities: colours (1)

- The above all assumes a *monochromatic* light source
- Light of different colours does not interfere and so each colour creates its own fringes
- Fringe separation is proportional to wavelength and so red fringes are bigger than blue fringes
- Central fringe coincides in all cases

Practicalities: colours (2)



Observe bright central fringe (white with coloured edges) surrounded by a complex pattern of colours. Makes central fringe easy to identify!

Interference (1)

- It is easy to calculate the positions of maxima and minima, but what happens between them?
- Explicitly sum the amplitudes of the waves $A=\cos(kx_1-\omega t-\phi)+\cos(kx_2-\omega t-\phi)$
- Write $x_1=x-\delta/2$, $x_2=x+\delta/2$ and use $\cos(P+Q)+\cos(P-Q)=2\cos(P)\cos(Q)$
- Simplifies to $A=2\cos(kx-\omega t-\phi)\times\cos(k\delta/2)$

Interference (2)

- Amplitude is $A=2\cos(kx-\omega t-\phi)\times\cos(k\delta/2)$
- Intensity goes as square of amplitude so $I=4\cos^2(kx-\omega t-\phi)\times\cos^2(k\delta/2)$
- First term is a rapid oscillation at the frequency of the light; all the interest is in the second term
- $I=\cos^2(k\delta/2)=[1+\cos(k\delta)]/2$

Interference (3)

- Intensity is $I=\cos^2(k\delta/2)=[1+\cos(k\delta)]/2$
- Intensity oscillates with maxima at $k\delta=2n\pi$ and minima at $k\delta=(2n+1)\pi$
- Path length difference is $\delta=dy/D$
- Maxima at $y=D\delta/d$ with $\delta=2n\pi/k$ and $k=2\pi/\lambda$ giving $y=n\lambda D/d$

Exponential waves (1)

- Whenever you see a cosine you should consider converting it to an exponential! $\exp(ix)=\cos(x)+ i \sin(x)$
- Basic wave in exponential form is $\cos(kx-\omega t-\phi)=\mathrm{Re}\{\exp[i(kx-\omega t-\phi)]\}$
- Do the calculations in exponential form and convert back to trig functions at the very end

Exponential waves (2)

- Repeat the interference calculation
- Explicitly sum the amplitudes of the waves
  $A=\mathrm{Re}\{\exp[i(kx-k\delta/2-\omega t-\phi)]+\exp[i(kx+k\delta/2-\omega t-\phi)]\}$
  $=\mathrm{Re}\{\exp[i(kx-\omega t-\phi)]\times(\exp[-ik\delta/2]+\exp[+ik\delta/2])\}$
  $=\mathrm{Re}\{\exp[i(kx-\omega t-\phi)]\times 2\cos[k\delta/2]\}$
  $=2\cos(kx-\omega t-\phi)\times\cos[k\delta/2]\}$
- Same result as before (of course!) but can be a bit simpler to calculate
- Will use complex waves where convenient from now on

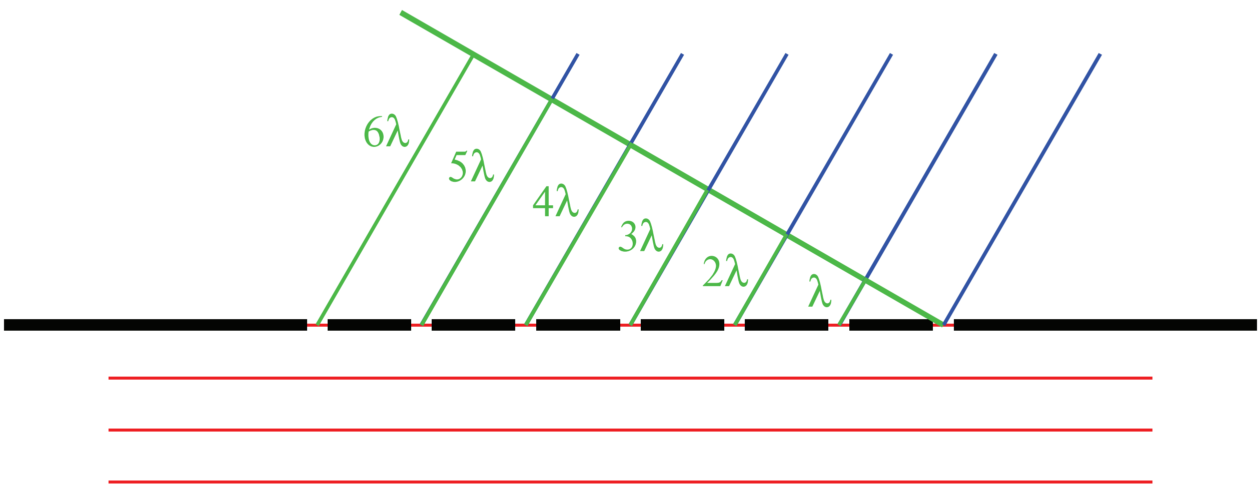
Exponential waves (4)

- Use a complex amplitude to represent the wave
  $A=\exp[i(kx-\omega t-\phi+k\delta/2)]\times 2\cos[k\delta/2]$
- The intensity of the light is then given by the square modulus of the amplitude:
  $I=A^*A=4\cos^2[k\delta/2]$
- This approach loses the rapid time oscillations, but we have previously ignored these anyway! Result is the peak intensity which is twice the average intensity.

Diffraction gratings (1)

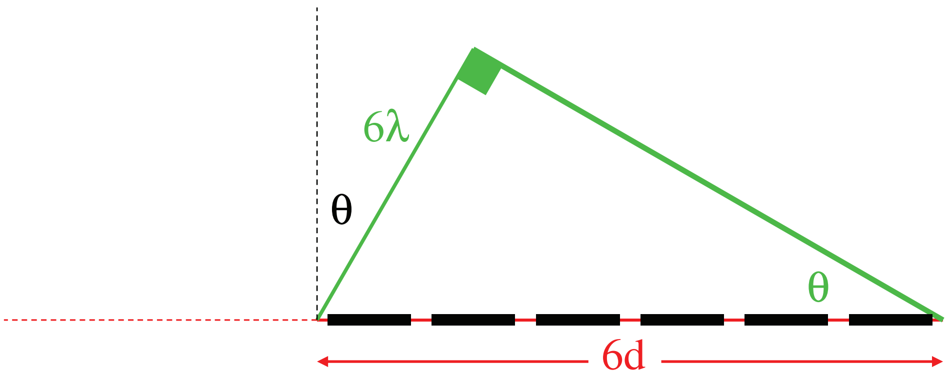
- A diffraction grating is an extension of a double slit experiment to a very large number of slits
- Gratings can work in transmission or reflection but we will only consider transmission gratings
- The basic properties are easily understood from simple sketches, and most of the advanced properties are (in principle!) off-syllabus

Diffraction gratings (7)



Points on a wavefront must be in phase, so the extra distances travelled must be multiples of a wavelength

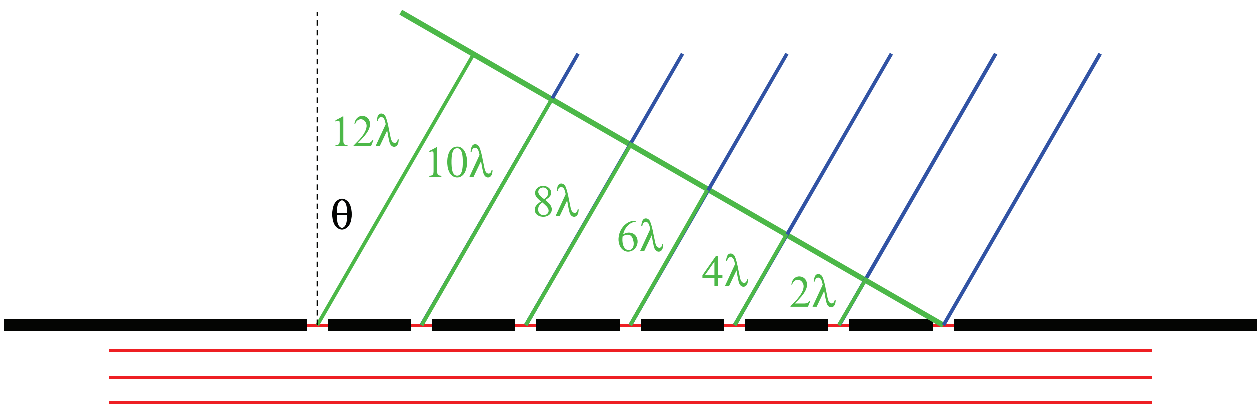
Diffraction gratings (8)



From trigonometry we see that $\sin(\theta)=6\lambda/6d$ where θ is the angle between the ray direction and the normal

Basic diffraction equation: $\lambda=d\sin(\theta)$

Diffraction gratings (9)

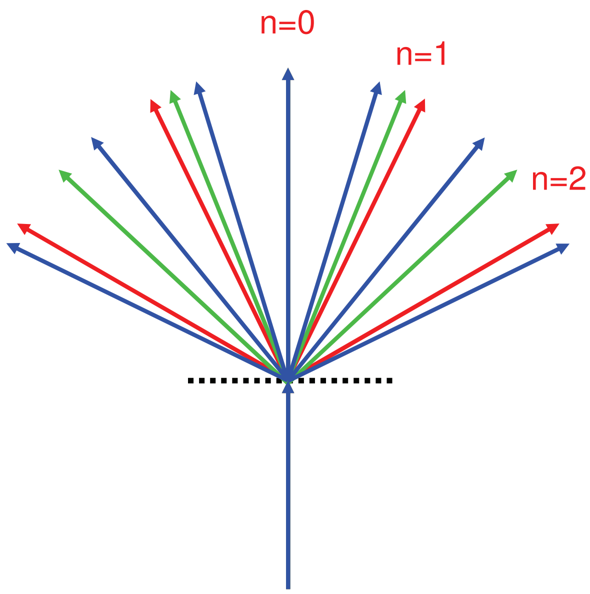


Can also get diffraction in the same direction if the wavelength is halved!

General diffraction equation: $n\lambda=d\sin(\theta)$

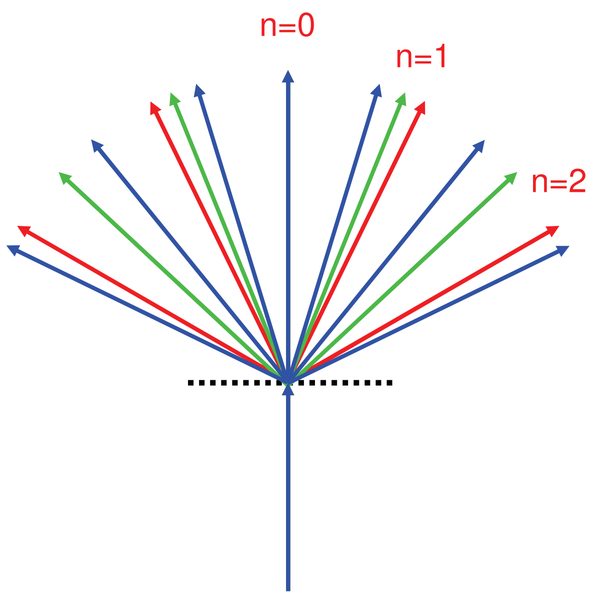
Dispersion (1)

- $n \lambda = d \sin(\theta)$
- Different colours have different wavelengths and so diffract at different angles
- Take red (λ=650nm), green (λ=550nm), and blue (λ=450nm) light and d=1.5μm

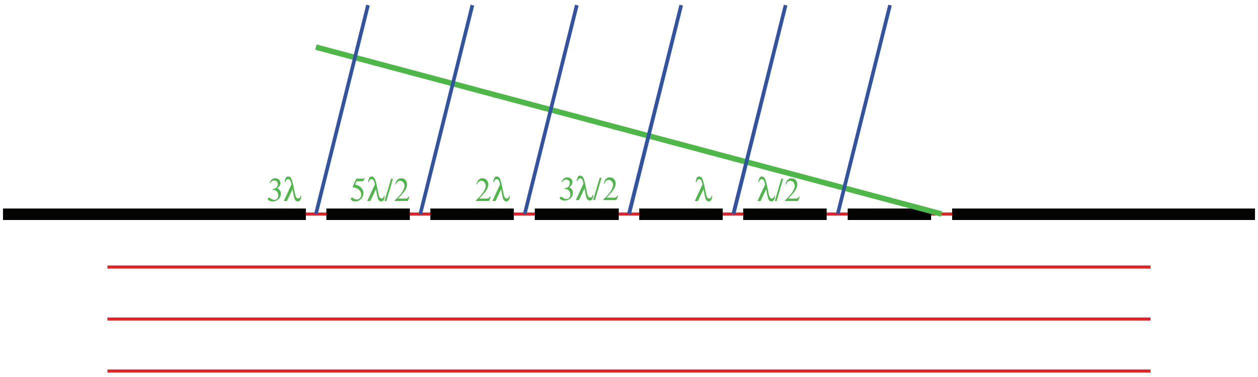


Dispersion (2)

- Angular dispersion of a grating is $D = d\theta/d\lambda$ $= n/d \cos(\theta)$ $= n/\sqrt{(d^2 - n^2\lambda^2)}$
- Increases with *order* of the spectrum and when $d \approx n\lambda$
- Note that high order spectra can overlap!

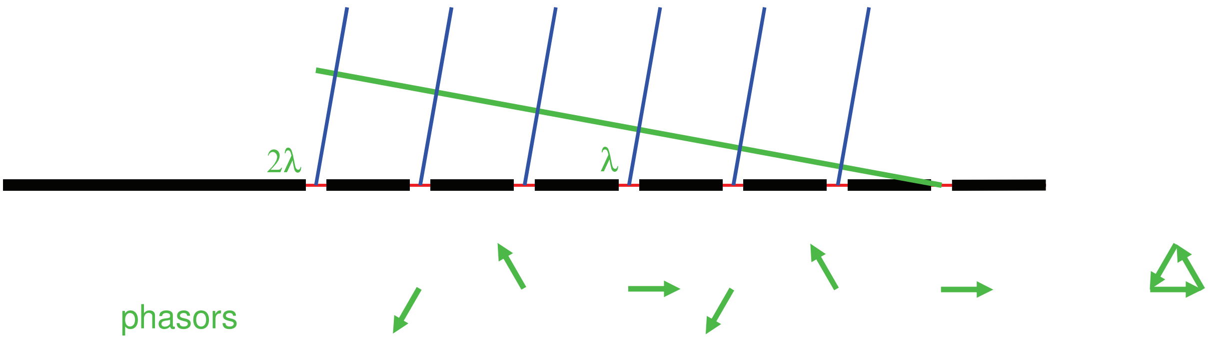


Between the peaks (1)



We also need to work out what happens in other directions. Between n=0 and n=1 there is a direction where the light from adjacent sources exactly cancel each other out – just as for two slits!

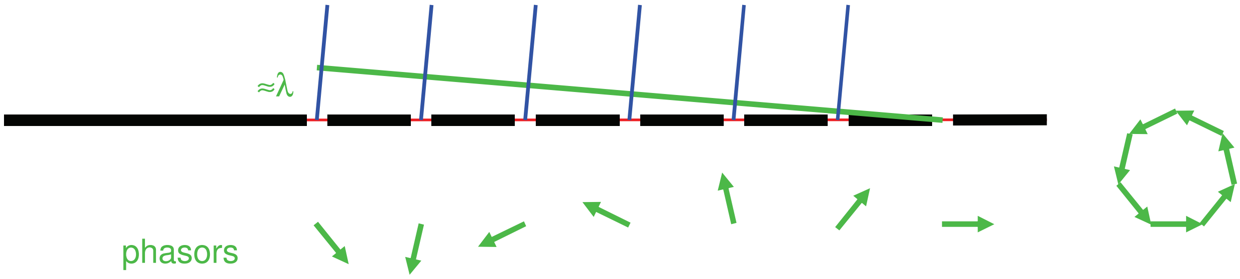
Between the peaks (2)



Closer to n=0 there is a direction where light from groups of three adjacent sources exactly cancels out

With an *infinite* number of sources will get cancellation in *every* direction except for the main peaks!

Between the peaks (3)



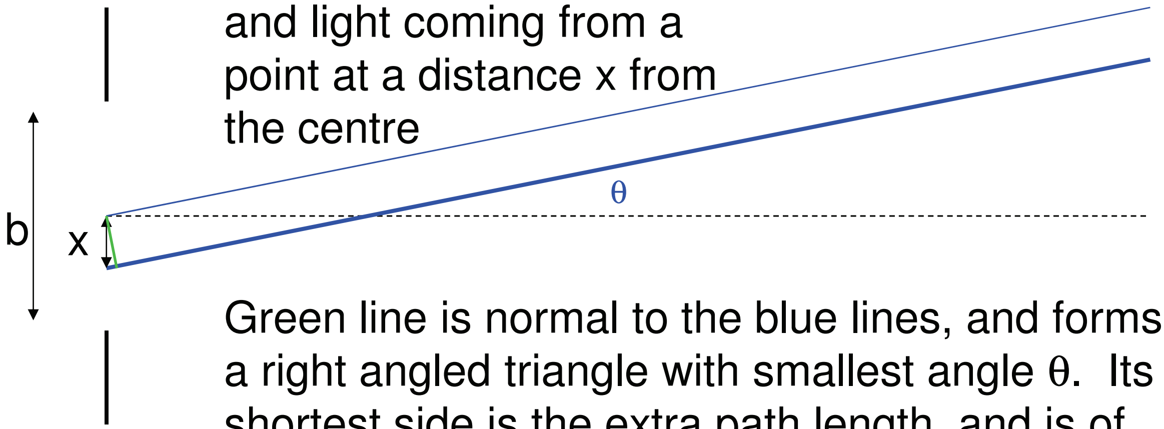
With an *finite* number of sources the last effective cancellation will occur when the path length difference between the first and last sources is about λ

Single slit diffraction (1)

- Consider a single slit illuminated by a plane wave source
- If the slit is perfectly narrow this will create a cylindrical wave
- If the slit has significant width we must treat it as a continuous distribution of sources and integrate over them

Single slit diffraction (2)

Consider a slit of width b and light coming from a point at a distance x from the centre



Green line is normal to the blue lines, and forms a right angled triangle with smallest angle θ. Its shortest side is the extra path length, and is of size $\delta = x \sin(\theta)$

Light coming from x has an extra phase $k\delta$

Single slit diffraction (3)

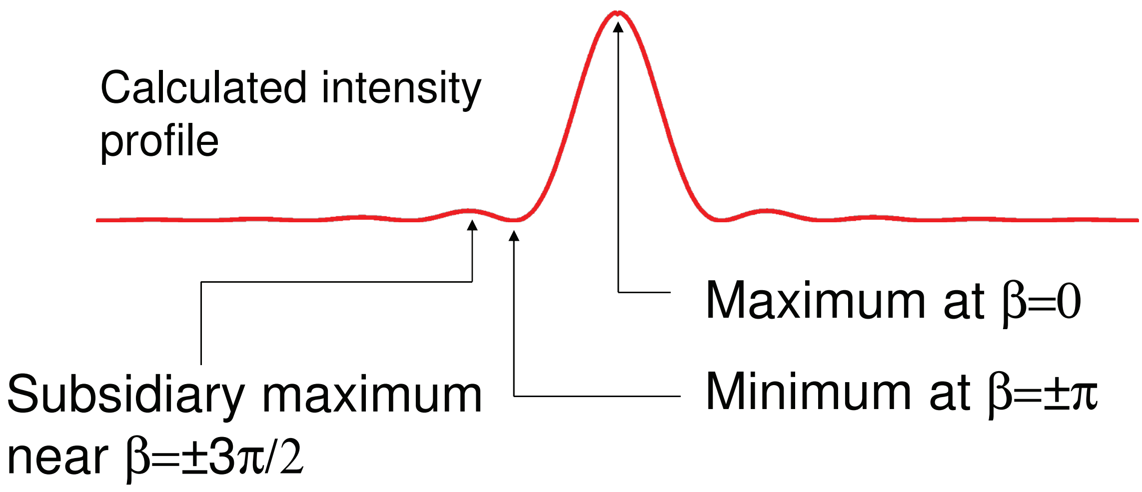
- Final amplitude is obtained by integrating over all points x in the slit and combining the phases, most simply in complex form
- $A = \int \exp(ik\delta)\, dx = \int \exp(ik\, x \sin(\theta))\, dx$
- Integral runs from $x=-b/2$ to $x=b/2$ and then normalise by dividing by b

Single slit diffraction (4)

- $A = \sin[(kb/2)\sin(\theta)]/[(kb/2)\sin(\theta)]$
- $A = \sin(\beta)/\beta$ with $\beta = (kb/2)\sin(\theta)]$

- Light intensity goes as the square of A
- $I = I_0[\sin(\beta)/\beta]^2$
- $I = I_0 \text{sinc}^2(\beta)$

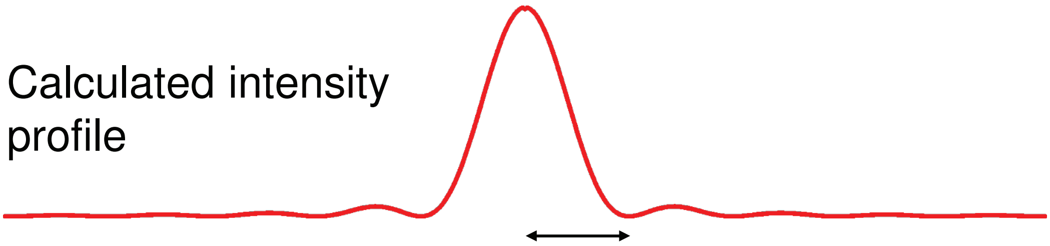
Single slit diffraction (5)



Width defined by minima at $\beta = (kb/2)\sin(\theta) = \pm\pi$

Single slit diffraction (6)

Calculated intensity profile

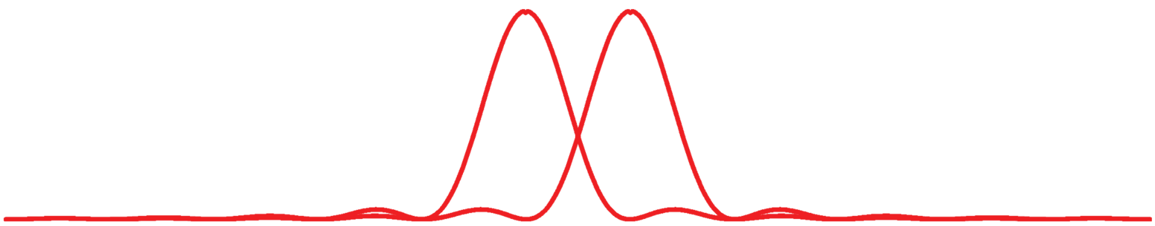


Width defined by minima at $\beta=(kb/2)\sin(\theta)=\pm\pi$

Solution is $\theta=\arcsin(2\pi/bk)=\arcsin(\lambda/b)\approx\lambda/b$

Get significant diffraction effects when the slit is small compared to the wavelength of light!

Single slit diffraction (8)



- The *Rayleigh Criterion* says that two diffraction limited images are well resolved when the maximum of one coincides with the first minimum of the other, so $\theta_R=\arcsin(\lambda/b)$
- Alternative criteria can also be used

Resolving Power

- Basic ideas for the resolving power of a diffraction grating have been seen already, but now we can do the calculation properly
- Resolving power is defined as the reciprocal of the smallest change in wavelength which can be resolved as a fraction of the wavelength

Dispersion

- $n\lambda = d\sin(\theta)$
- Dispersion described by $n\, d\lambda/d\theta = d\cos(\theta)$
- So the limiting wavelength resolution depends on the limiting angular resolution according to $n\,\delta\lambda = d\cos(\theta)\,\delta\theta$