CP2: Optics

Jonathan Jones

Part 1: Geometric Optics

Why study optics?

- History
- Technology
- Simplicity
- Centrality

- Passing CP2

CPSC 4289

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 8 June 2010, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a fresh page.

The use of calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

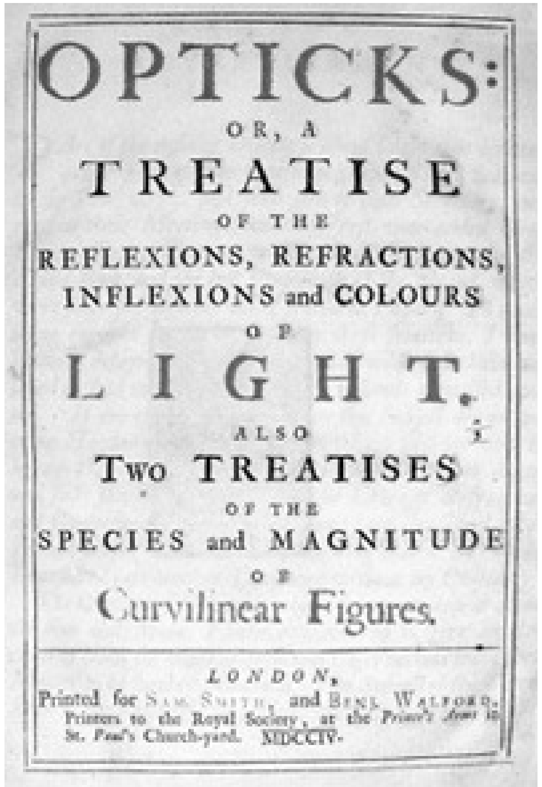
Do NOT turn over until told that you may do so.

1

The problem of teaching optics

- Some feedback comments from 2011-12
  1. Too much A-Level content
  2. This was not a basic intro to the course! I hadn't studied optics before and found all the work far too advanced for me to understand.
- Can’t keep everyone happy! This course is aimed squarely at beginners but does assume knowledge of the absolute basics

Optics around 1700



OPTICKS:

OR, A

TREATISE

OF THE

REFLEXIONS, REFRACTIONS,
INFLEXIONS and COLOURS

OF

LIGHT.

ALSO

Two TREATISES

OF THE

SPECIES and MAGNITUDE

OF

Curvilinear Figures.

LONDON,
Printed for SAM. SMITH, and BENJ. WALFORD,
Printers to the Royal Society, at the *Prince's Arms* in
St. *Paul's* Church-yard. MDCCIV.

- Lots of facts known about light
- Little understood about the underlying principles
- Newton making trouble as usual

- Waves or particles?

Waves or particles?

- Light travels in straight lines
  - Waves travel in circles (chuck a rock in a pond and watch the ripples spread out)
  - But particles in crossed beams would collide?
- Light reflects off mirrors and leaves at the same angle as it came in
  - Makes sense for particles (conservation of momentum)

Waves or particles?

- Light bends (refracts) when moving between different media

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

  – Newton had a semi-plausible explanation for particles
  – Easy to explain for waves *if* they travel in straight lines!

Waves or particles?

- Diffraction effects not really understood
  - Newton’s rings provide excellent evidence for wave behaviour, but Newton was unhappy with the wave model
- Underlying basis of colour hardly understood at all
- Polarization only recently discovered (Iceland Spar)

Huygens's Problem

- For I do not find that any one has yet given a probable explanation of … why it is not propagated except in straight lines, and how visible rays … cross one another without hindering one another in any way.
- *Christian Huygens "Treatise on Light" translated by Silvanus P. Thompson http://www.gutenberg.org/etext/14725*

Huygens's Principle

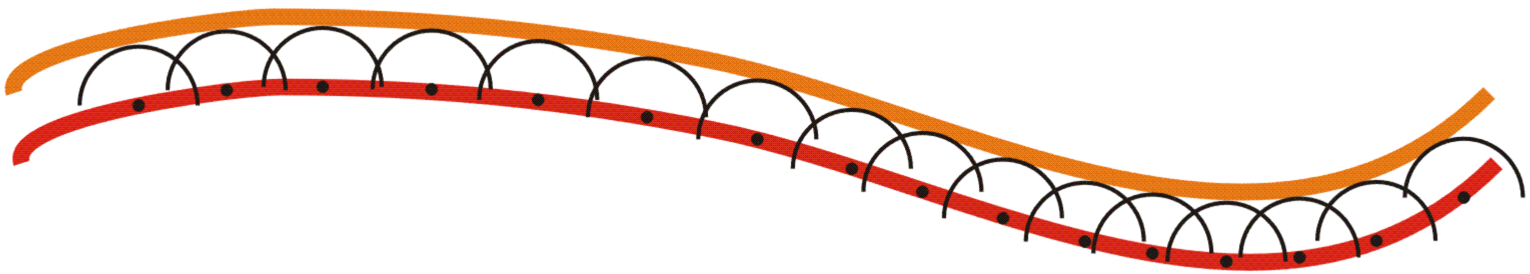
- Huygens's principle tells us to consider each point on a wavefront as a new source of radiation and add the "radiation" from all of the new "sources" together. Physically this makes *no* sense at all. Light does not emit light; only accelerating charges emit light. Thus we will begin by throwing out Huygens's principle completely; later we will see that it actually does give the right answer for the wrong reasons. *(Melvin Schwartz, Principles of Electrodynamics)*

Huygens's Model

- Light is made up of a series of pulsations in the ether, an otherwise undectable substance filling all space
- Each pulsation causes a chain of secondary pulsations to spread out ahead
- In certain directions these pulsations reinforce one another, creating an intense pulsation that appears as visible light

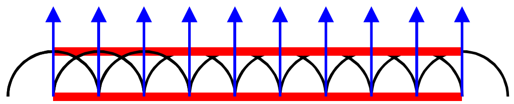
Huygens's Construction

- Every point on a wavefront may be regarded as a source of secondary wavelets which spread out with the wave velocity.
- The new wavefront is the envelope of these secondary wavelets.



Straight lines

- Straight wavefronts *stay* straight



- Points on the wavefront all move forward at the same speed in a direction normal to the wavefront. *All points on a wavefront correspond to the same point in time*.
- Light rays travel along these normals

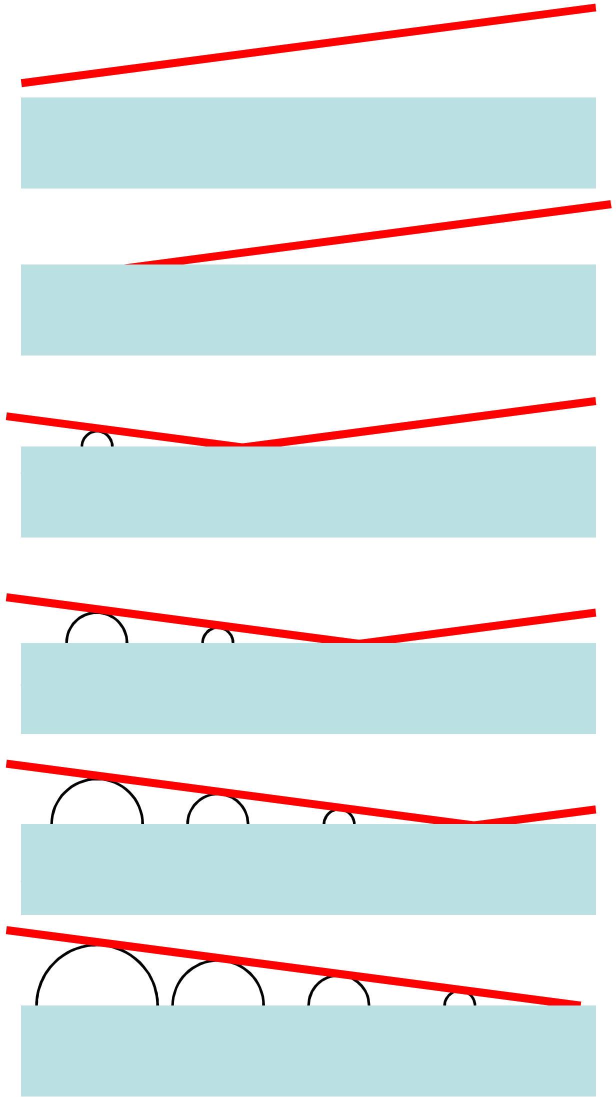
Problems

1. Why do wavefronts travel forwards and not backwards?
2. What happens at the edges?

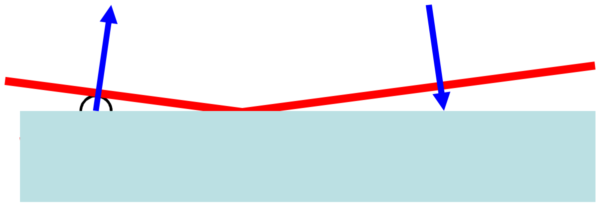
- These questions can be answered with a more serious model but that is largely beyond the scope of this course.

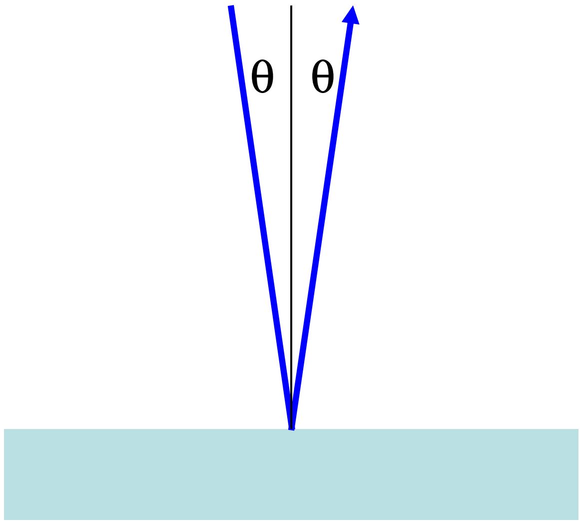
Reflection

- Wavefront propagates in a straight line
- As it hits the surface it becomes a source of secondary wavelets
- Wavelets all “grow” at the same speed
- Envelope of these forms new wavefront



Reflection

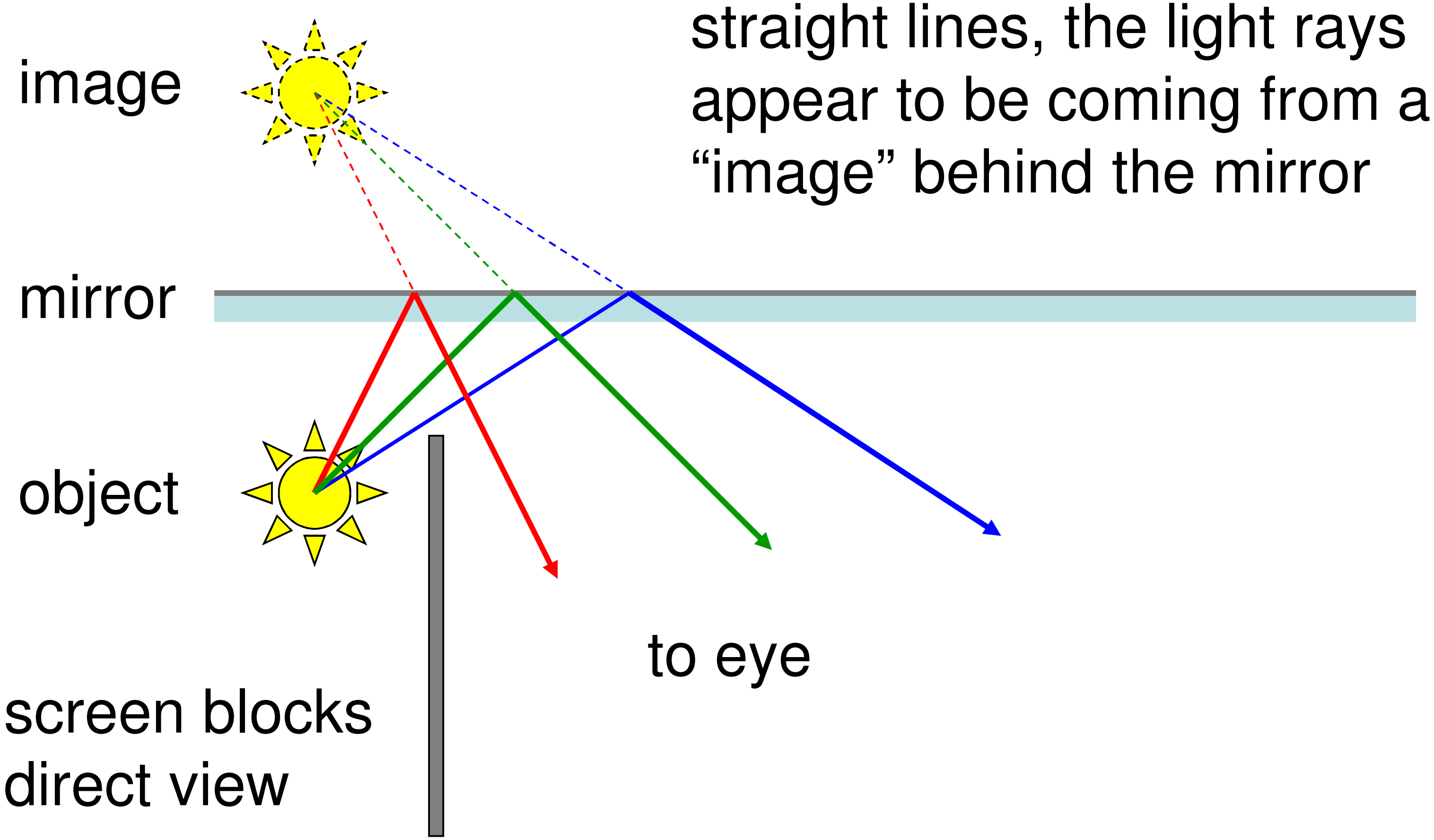




- Reflected ray is at the same angle as incident ray
- Reflected wavefront is at the same angle as incident wavefront
- Occurs because the secondary wavelets grow at the same rate in both wavefronts

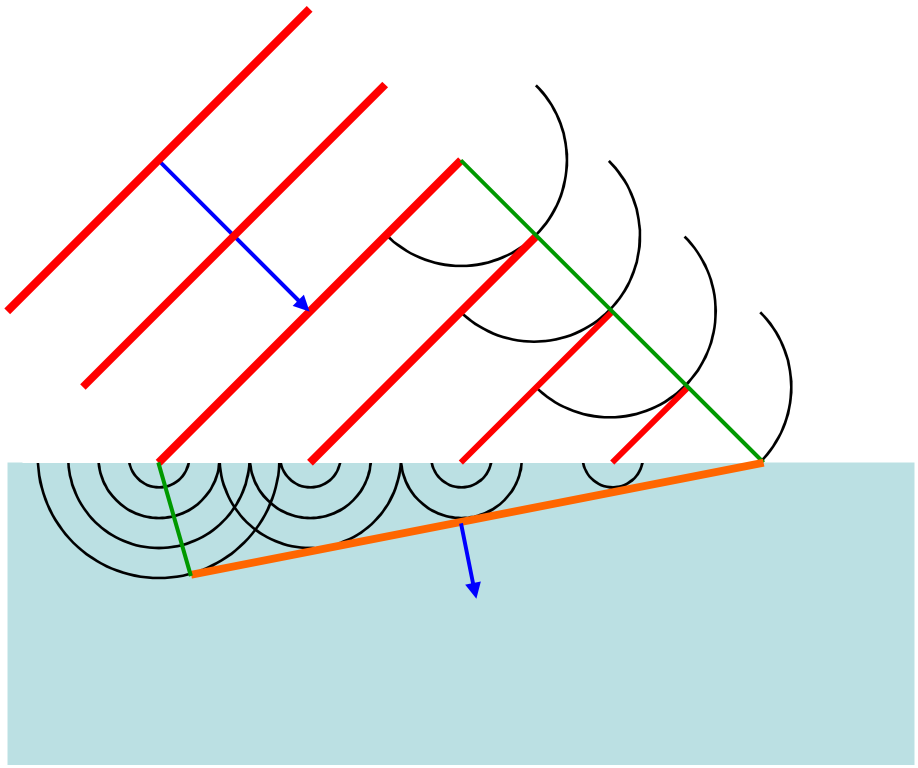
Image in a mirror

Since light normally travels in straight lines, the light rays appear to be coming from an "image" behind the mirror



Refraction

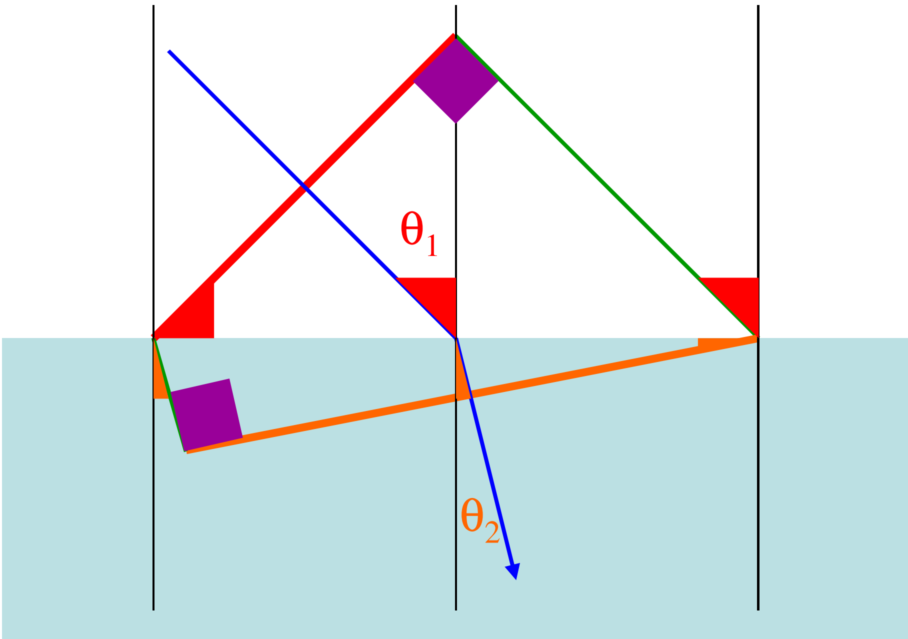
- Refraction is easily explained if wavelets travel more slowly in glass than in air



The two green lines are both four wavelets long. The start points of each line are points on a wavefront and so the end points must also be corresponding points on the new wavefront

Refraction

- Refraction is easily explained if wavelets travel more slowly in glass than in air



•Cut diagram down to essentials.

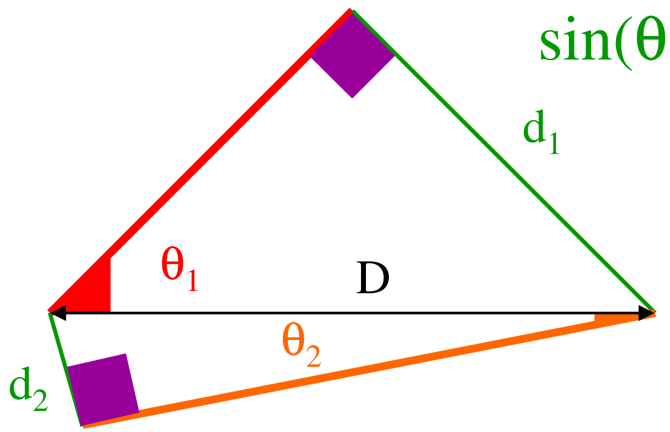
•Add construction lines and rays

•Note common angles

Refraction

- The light ray takes the same length of time to travel along the two green paths
- Travels at different speeds: $v=c/n$, where n is the refractive index

$$\sin(\theta_1)=d_1/D$$



$$\sin(\theta_2)=d_2/D$$

$$d_1/v_1 = d_2/v_2$$

$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$

Snell’s law of refraction

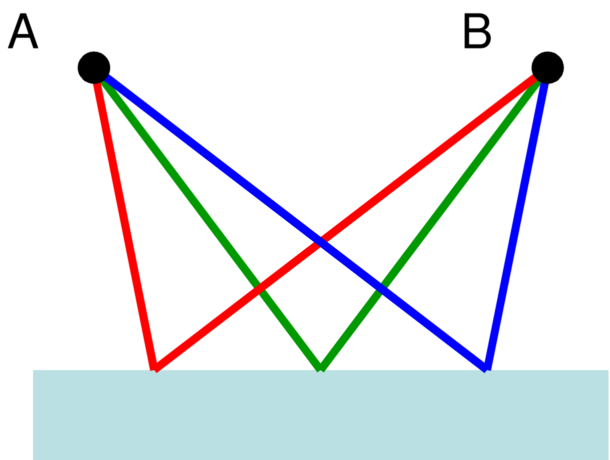
Some materials

Fermat’s Principle

- Fermat’s Principle of Least Time says that the path adopted by a light ray between any two points is the path that takes the smallest time
- Huygen’s model or ideas such as QED can be used to show that the path must be a local extremum (minimum, maximum, or inflection)
- Basic ideas probably known by Hero of Alexandria and by Alhacen (Ibn al-Haytham)
- Similar ideas will be seen in mechanics!

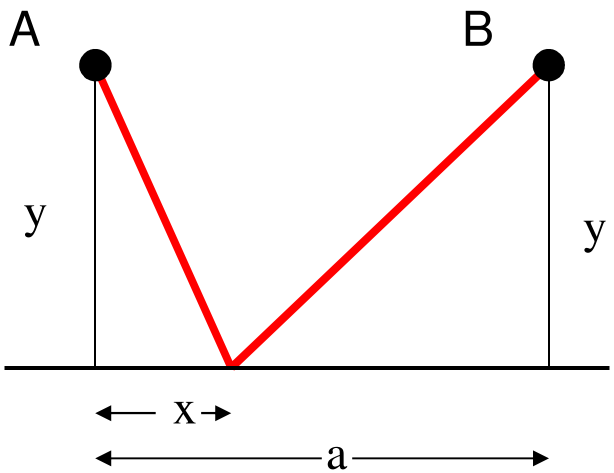
Reflection (Fermat)

A light ray takes the shortest (*least time*) path between two points



- At constant speed least time is equivalent to shortest distance
- Consistent with light moving in straight lines
- The green line is shorter than the red and blue lines
- Shortest path between A and B via the mirror!

Reflection (Fermat)



- Need to minimise total distance

$$s = \sqrt{y^2 + x^2} + \sqrt{y^2 + (a-x)^2}$$

$$\frac{\mathrm{d}s}{\mathrm{d}x} = \tfrac{1}{2}\left(y^2 + x^2\right)^{-1/2} \times 2x + \tfrac{1}{2}\left(y^2 + (a-x)^2\right)^{-1/2} \times 2(a-x)(-1) = 0$$

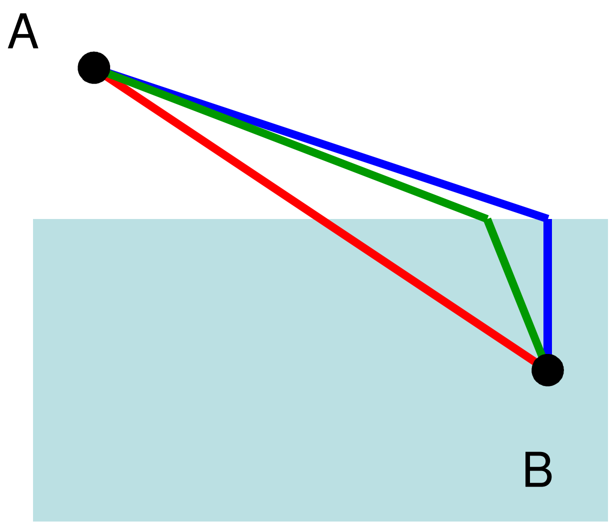
$$x = a - x$$

$$x = a/2$$

- Or use geometrical insight to spot that the answer is obvious if you reflect point B in the mirror.

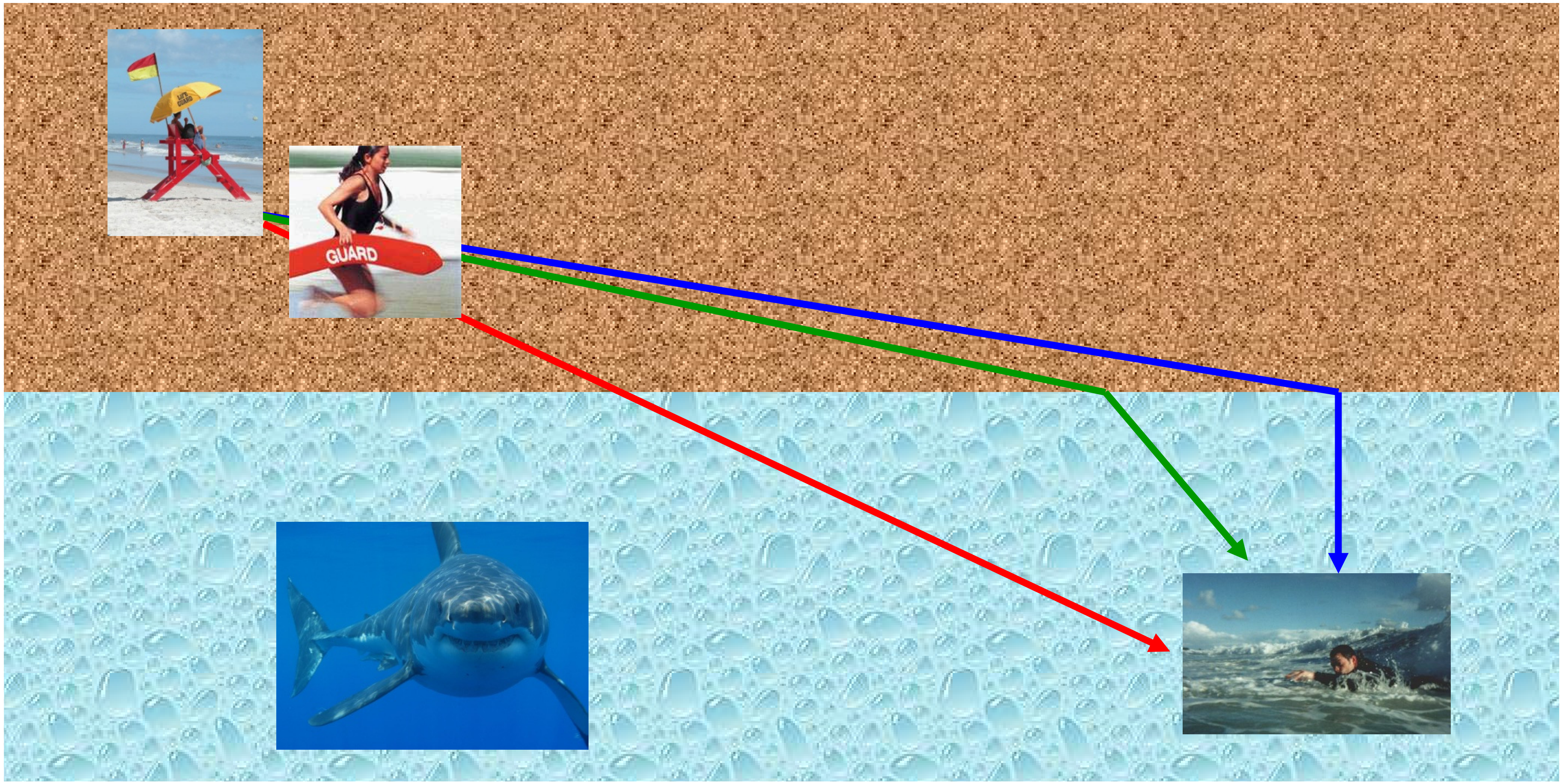
Refraction (Fermat)

A light ray takes the shortest (*least time*) path between two points



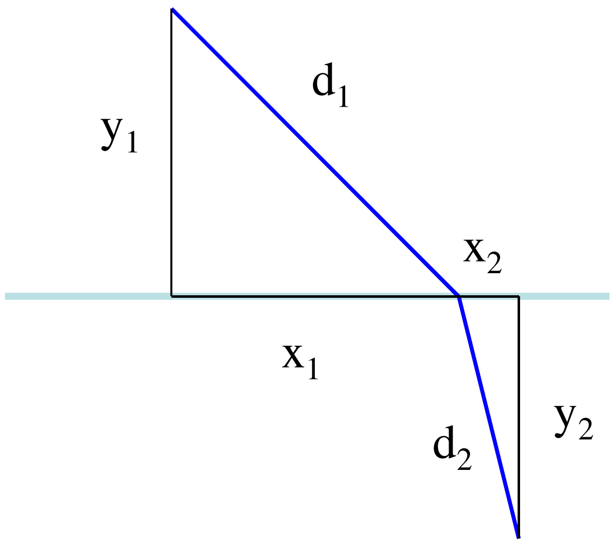
- At varying speed least time is *not* equivalent to shortest distance
- Light moves in straight lines in one medium but will bend at joins
- The green line is the *quickest* path between A and B!

Refraction (Fermat)



Refraction (Fermat)

- Minimise total time taken to travel along path



$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c}$$

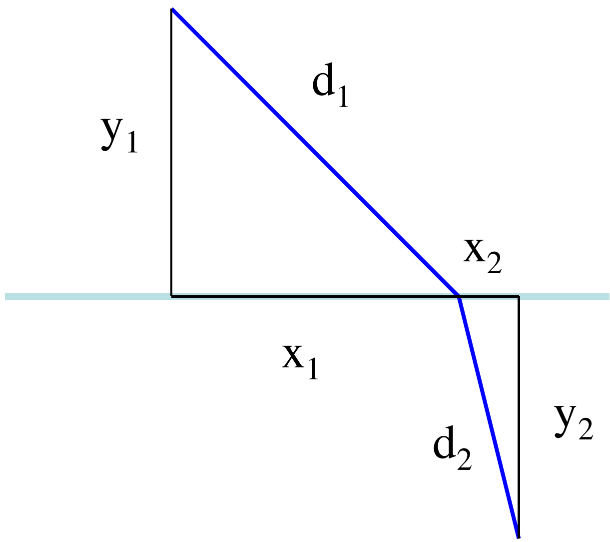
$$= \frac{n_1\sqrt{x_1^2 + y_1^2} + n_2\sqrt{x_2^2 + y_2^2}}{c}$$

Solve $dt/dx_1=0$

Refraction (Fermat)

$$\frac{\mathrm{d}t}{\mathrm{d}x_1} = \frac{n_1}{c} \times \tfrac{1}{2}\left(x_1^2 + y_1^2\right)^{-\frac{1}{2}} \times 2x_1$$

$$+ \frac{n_2}{c} \times \tfrac{1}{2}\left(x_2^2 + y_2^2\right)^{-\frac{1}{2}} \times 2x_2 \times (-1)$$

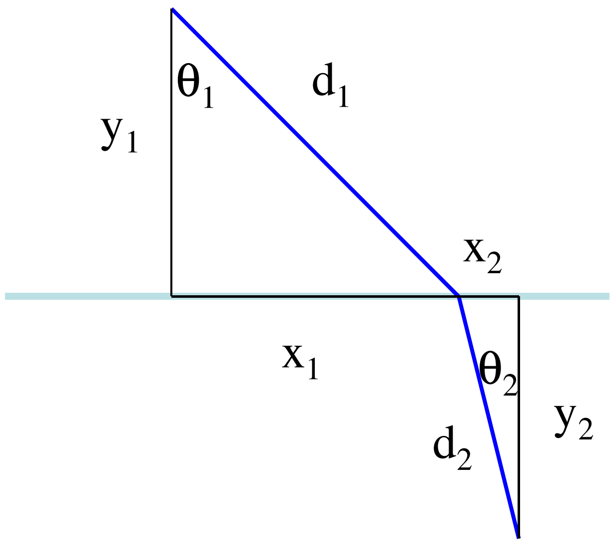


$$\frac{\mathrm{d}t}{\mathrm{d}x_1} = \frac{n_1 x_1}{c d_1} - \frac{n_2 x_2}{c d_2}$$

$$= \frac{n_1 \sin(\theta_1)}{c} - \frac{n_2 \sin(\theta_2)}{c}$$

Refraction (Fermat)

$$\frac{\mathrm{d}t}{\mathrm{d}x_1} = \frac{n_1 \sin(\theta_1) - n_2 \sin(\theta_2)}{c}$$

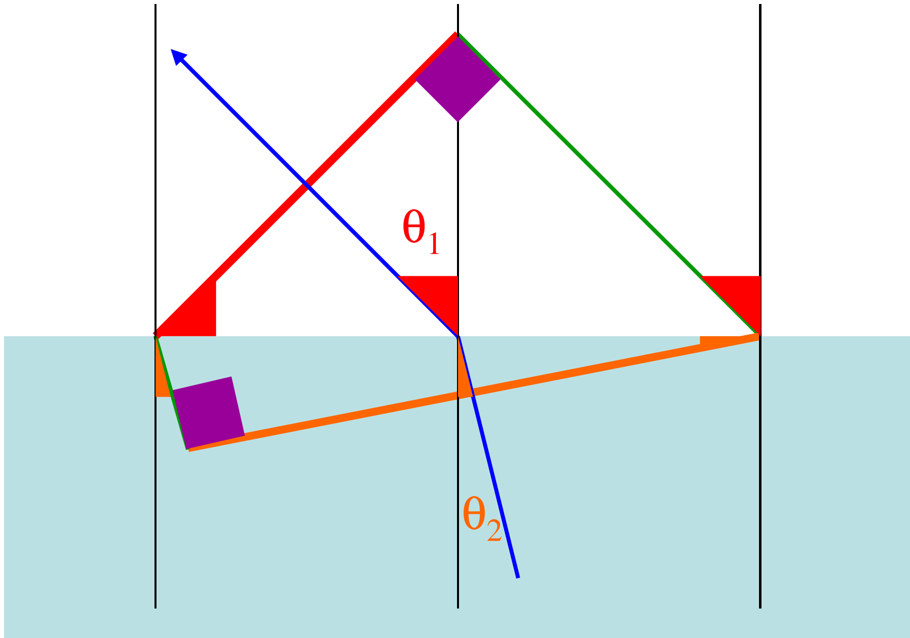


Solve $\dfrac{\mathrm{d}t}{\mathrm{d}x_1} = 0$ to get

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Reversibility

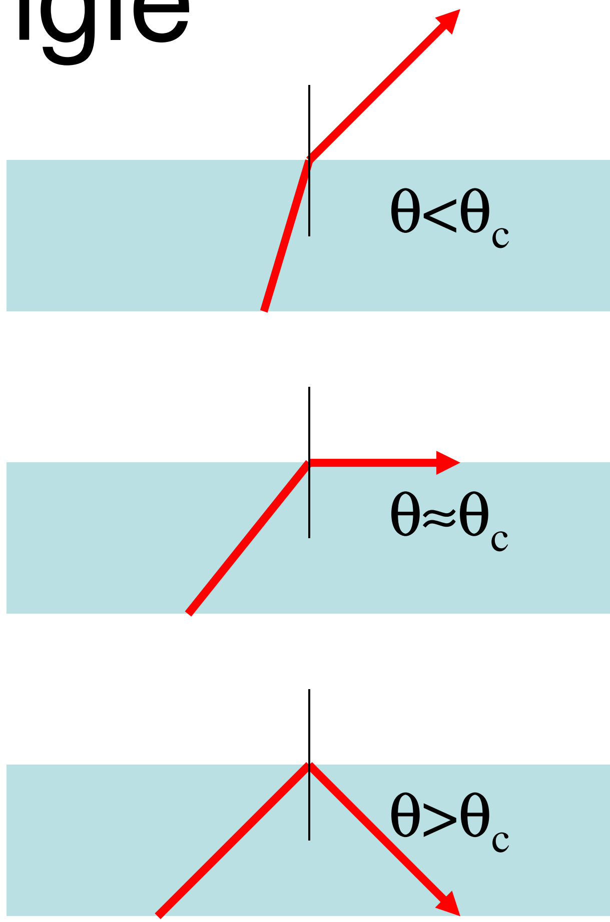
- Optical paths are always *reversible*



- A light ray travelling from glass into air will follow exactly the same path as a light ray travelling from air into glass, just in the opposite direction
- Obvious from Fermat

Critical angle

- A light ray travelling from a material with high refractive index to one with low refractive index is always bent away from the normal
- Angle is limited to 90º



Beyond the critical angle light ray undergoes *total internal reflection*

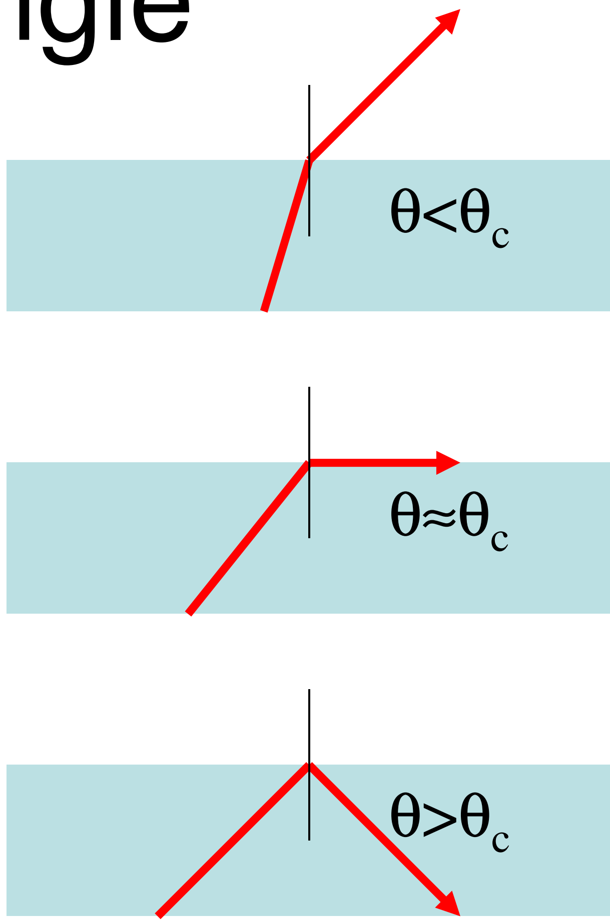
Critical angle

$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

At $\theta_1 = \theta_c$, $\theta_2 = 90^o$

$\sin(\theta_c) = n_2/n_1$

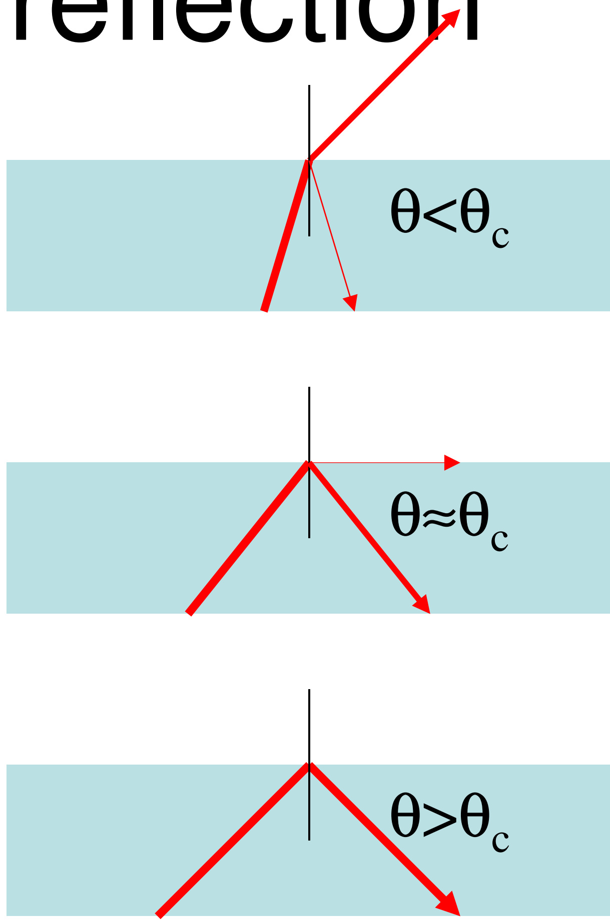
For glass to air
$\theta_c \approx \sin^{-1}(1/n)$



Beyond the critical angle light ray undergoes *total internal reflection*

Partial internal reflection

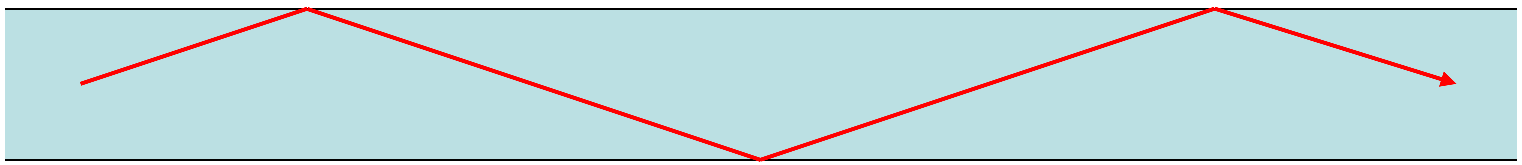
- For all angles less that the critical angle there is both a transmitted ray *and* a reflected ray
- Beyond critical angle light ray undergoes *total internal reflection*



The reflected ray is always reflected at the incident angle

Optic fibres (light pipes)

- Light can travel along an optic fibre by a series of total internal reflections
- If first reflection is beyond the critical angle then all reflections will be; the limit of transmission is set by the transparency of the glass

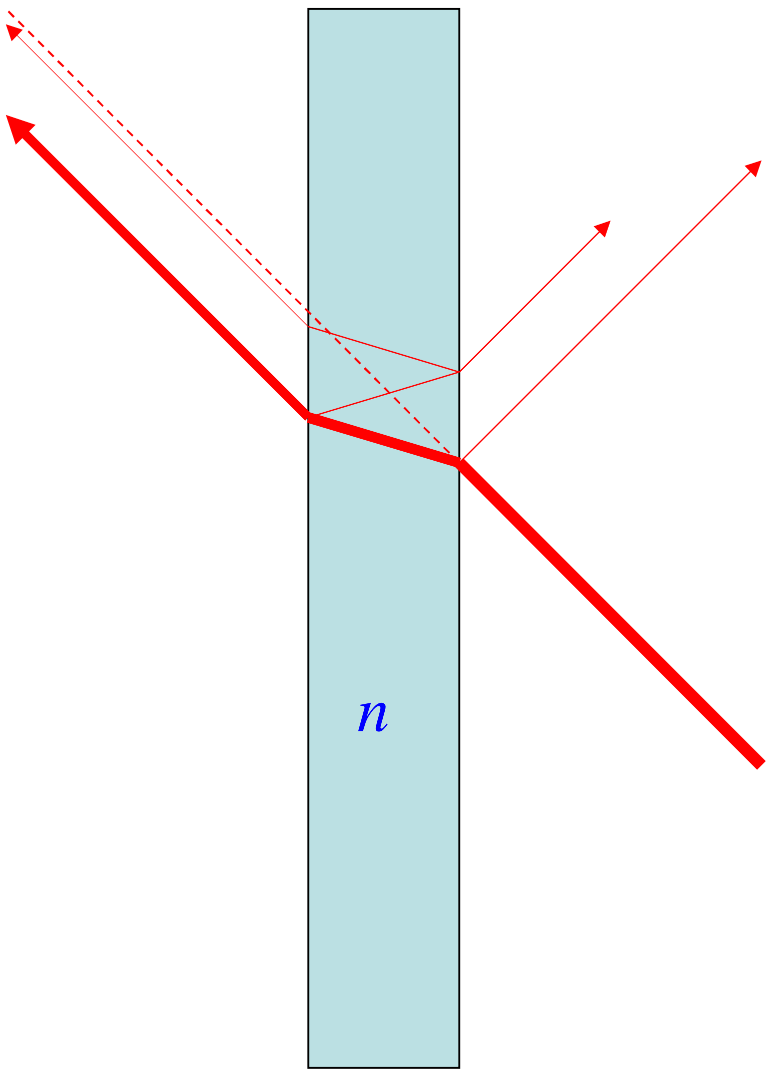


- Real fibres are made from two sorts of glass

Optic fibres (light pipes)

- This process will also work if the fibre is curved, as long as the radius of curvature is not too small
- Curves cause a small fraction of the light to leak out making the fibre visible

Pane of glass



- Light ray is refracted at both surfaces
- Ends up travelling in original direction but slightly offset
- Weak reflections at each surface
- Both reflections travel in same direction, but slightly offset

Sign conventions

- Once we switch from pictures to calculations we need a sign convention
- Sign conventions give rise to more confusion than any other topic, but fundamentally they are nothing more than a set of rules for choosing signs of distances in a consistent manner
- Similar problems occur in mechanics!

Sign conventions in mechanics

- The right way to do mechanics is to start by defining an axis system and then stick rigorously to this through the calculation
- For example we might put the y-axis pointing up so that distances *upwards* are *positive*. This means that the acceleration due to gravity must be *negative*

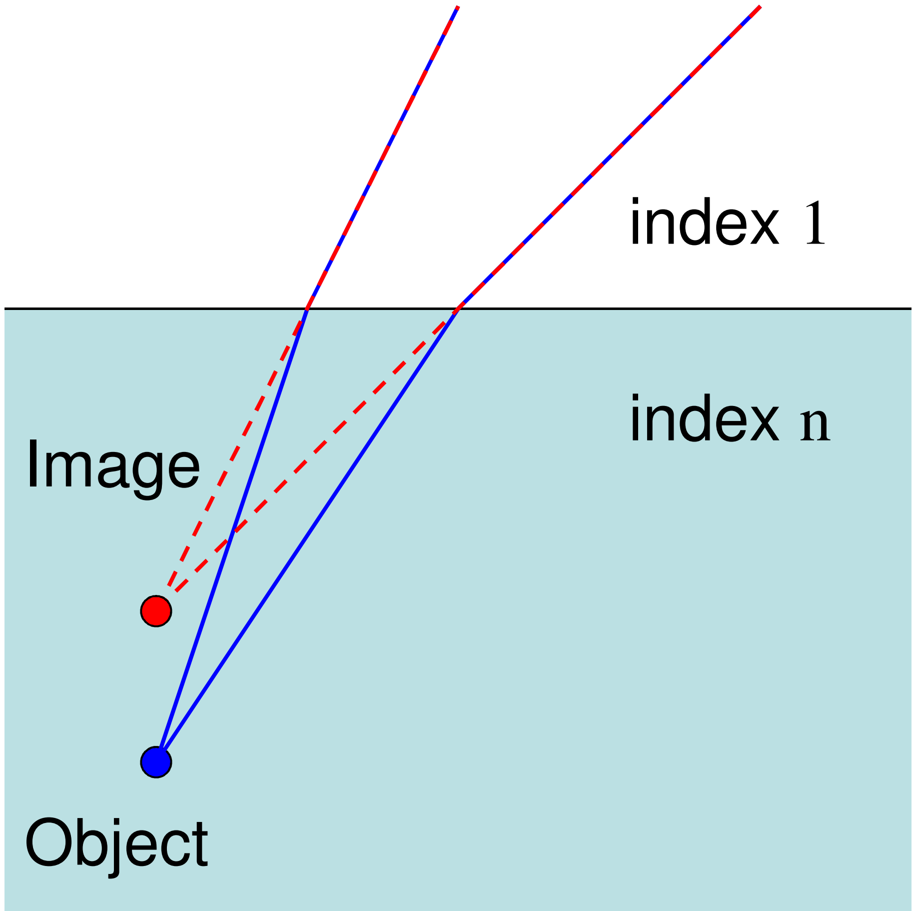
Sign conventions in mechanics

- For simple problems where we know roughly what the answer will be it is very tempting to fudge the axes and equations so that most things come out positive
- This works nicely for simple problems but can collapse in a messy heap when things get a bit more complicated

Sign conventions in optics

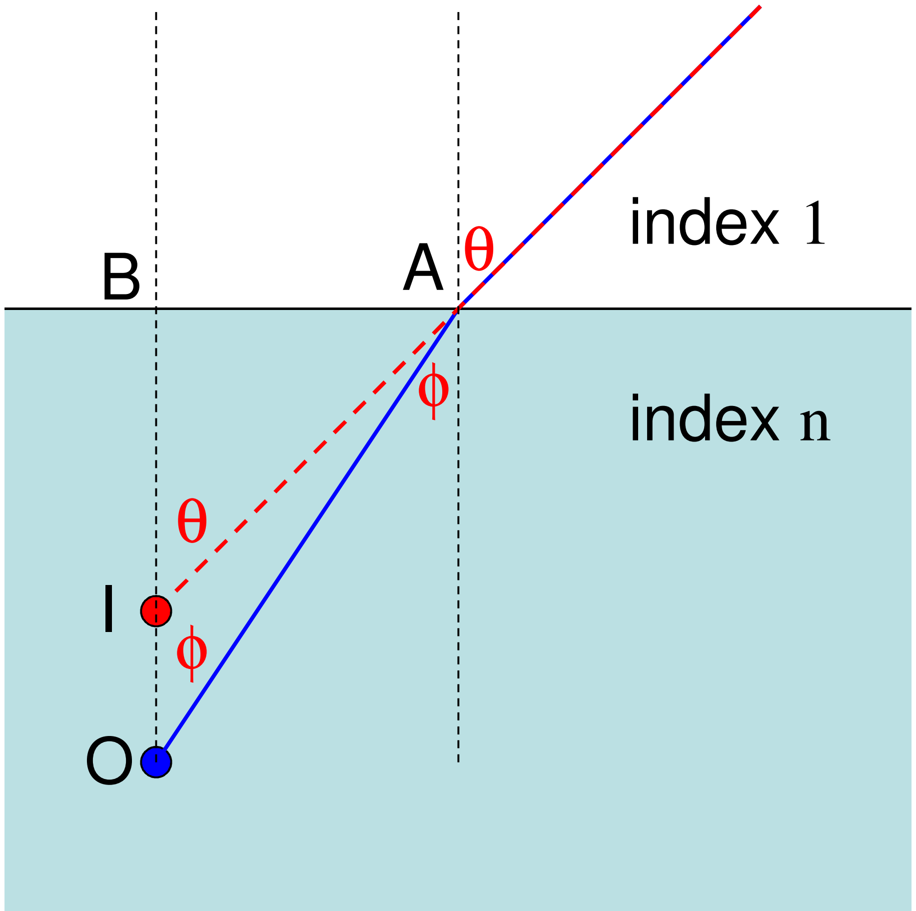
- In optics the “define an axis and stick to it” approach is called the *geometric sign convention*
- The “fudge things and hope for the best” approach is called the *real is positive sign convention*
- Following most books I start with the “real is positive” approach

Real and apparent depth



- If an underwater object is viewed from above it will appear to be in a different place from where it really is
- More on images later!
- Apparent depth is reduced by a factor of the refractive index *n*

Real and apparent depth



$n = \sin(\theta)/\sin(\phi)$

$\approx \tan(\theta)/\tan(\phi)$

$\tan(\theta) = AB/IB$

$\tan(\phi) = AB/OB$

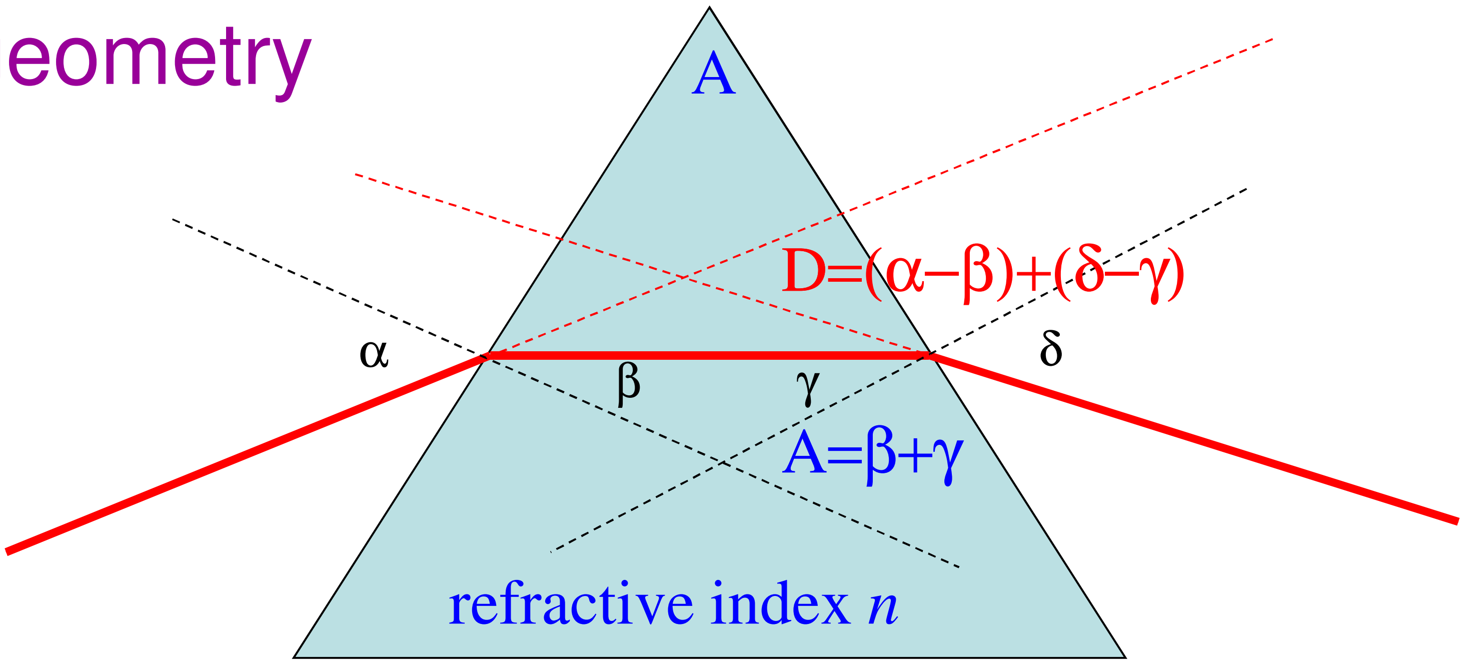
$n = (AB/IB)/(AB/OB)$

$= OB/IB$

All rays appear to come from point I at depth OB/n

Refraction at a prism

1) geometry

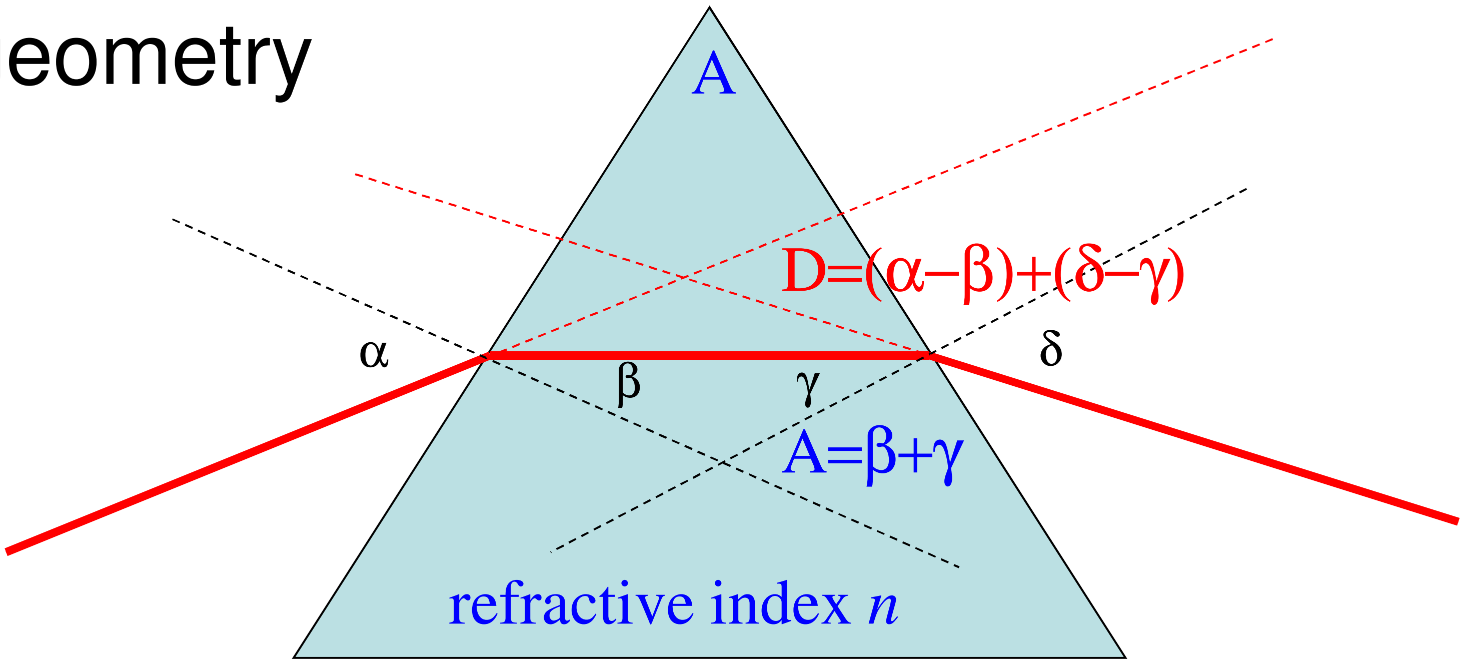


2) optics

$\sin(\alpha) = n \times \sin(\beta) \quad \sin(\delta) = n \times \sin(\gamma)$

Small angles

1) geometry



2) optics

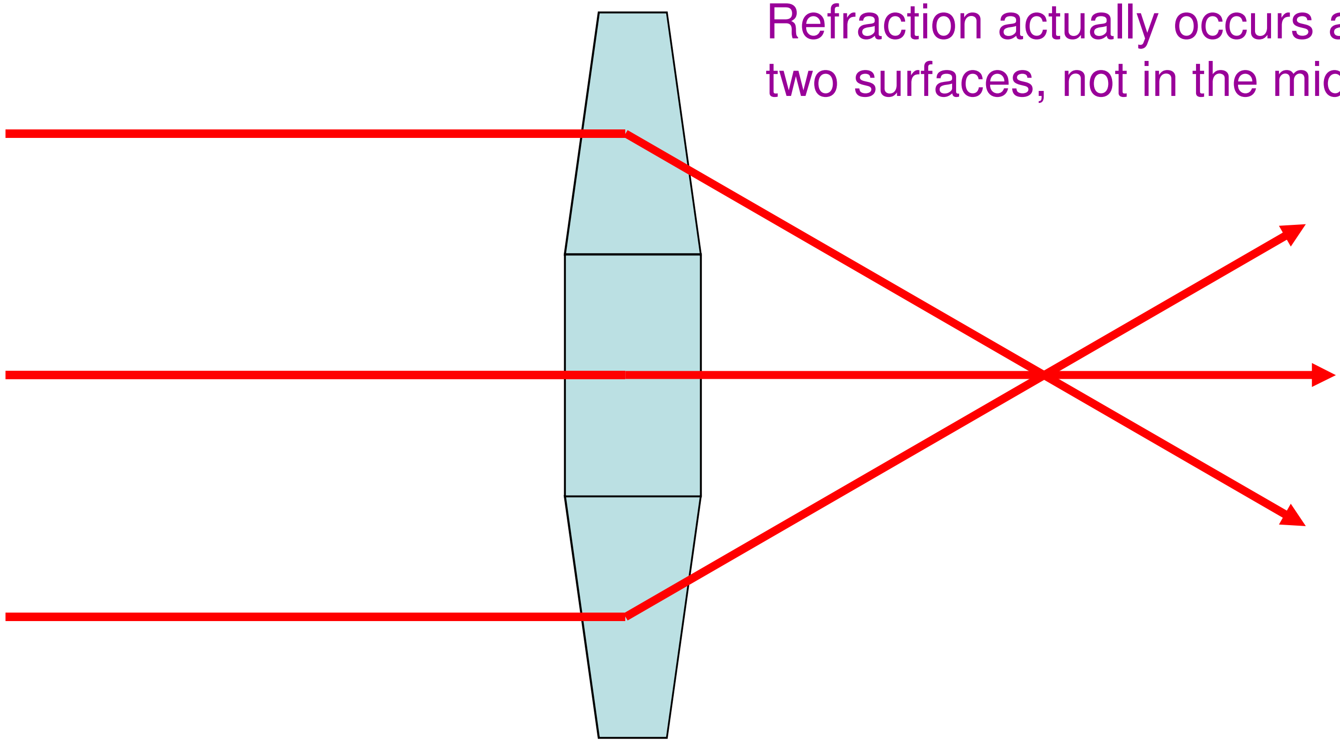
$\alpha = n \times \beta$ $\delta = n \times \gamma$

$$D = (n-1)\times\beta + (n-1)\times\gamma = (n-1)\times(\beta+\gamma) = (n-1)\times A$$

Dispersion

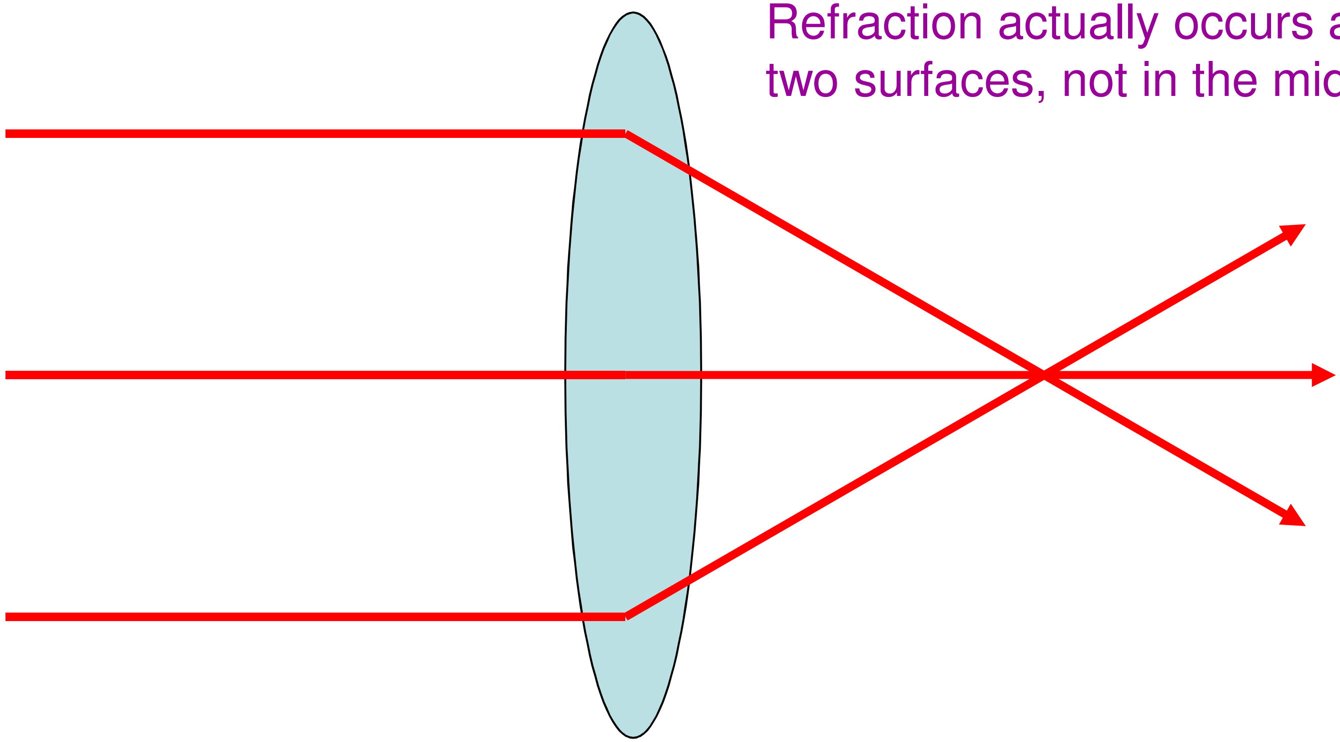
- Refractive index varies with frequency (dispersion). Usually n increases with frequency
- For visible light in glass $(n-1)$ typically increases by around 1–4% from red to blue
- Get an angular dispersion of about 1º
- Can be useful (for spectroscopy) or annoying (chromatic aberration)

Three thin prisms



A stack of three prisms will cause three parallel rays to meet at a single point (a focus)

A lens



Refraction actually occurs at the two surfaces, not in the middle!

A lens will cause three parallel rays to meet. The right shape will cause *all* parallel rays to meet.

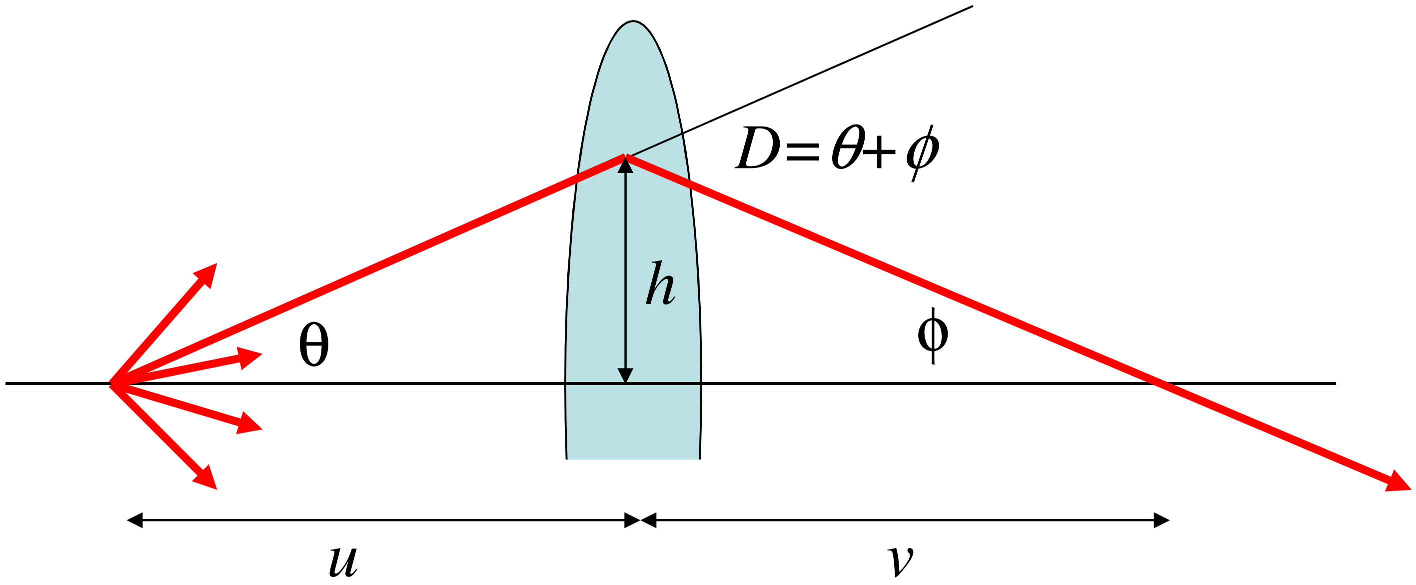
Approximations

"Geometrical optics is either very simple or very complicated". *Richard Feynman*

Paraxial approximation: only consider paraxial rays which lie very close to the optical axis and make small angles to it. This means that all important angles are small, and so we can assume that $\sin(\theta) \approx \tan(\theta) \approx \theta$.

Thin lens approximation: width of all lenses is small compared with other relevant distances, and so can be ignored

The lens formula (1)



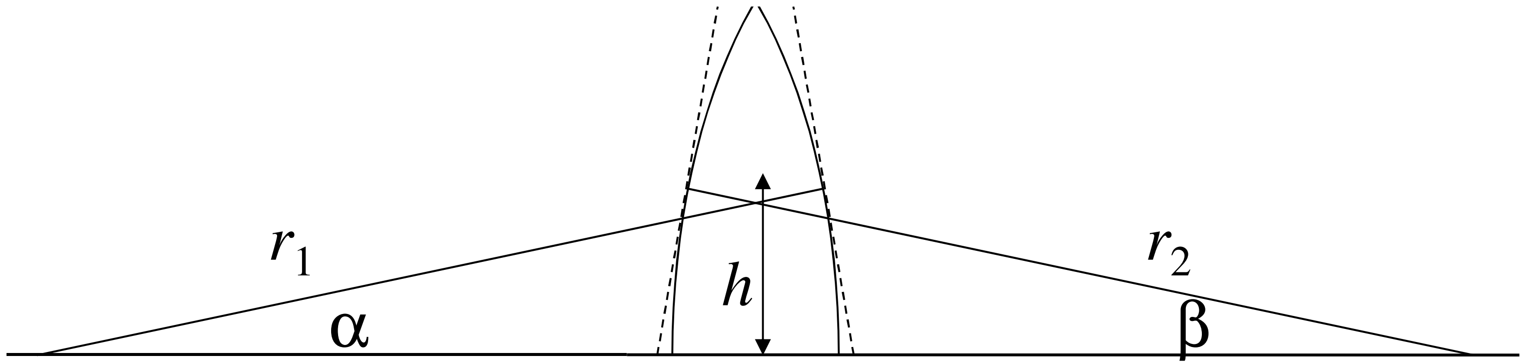
$$D = \theta + \phi = (n-1)\times A$$

$$\theta \approx h/u \quad \phi \approx h/v$$

rays focussed if

$$A \approx h/C$$

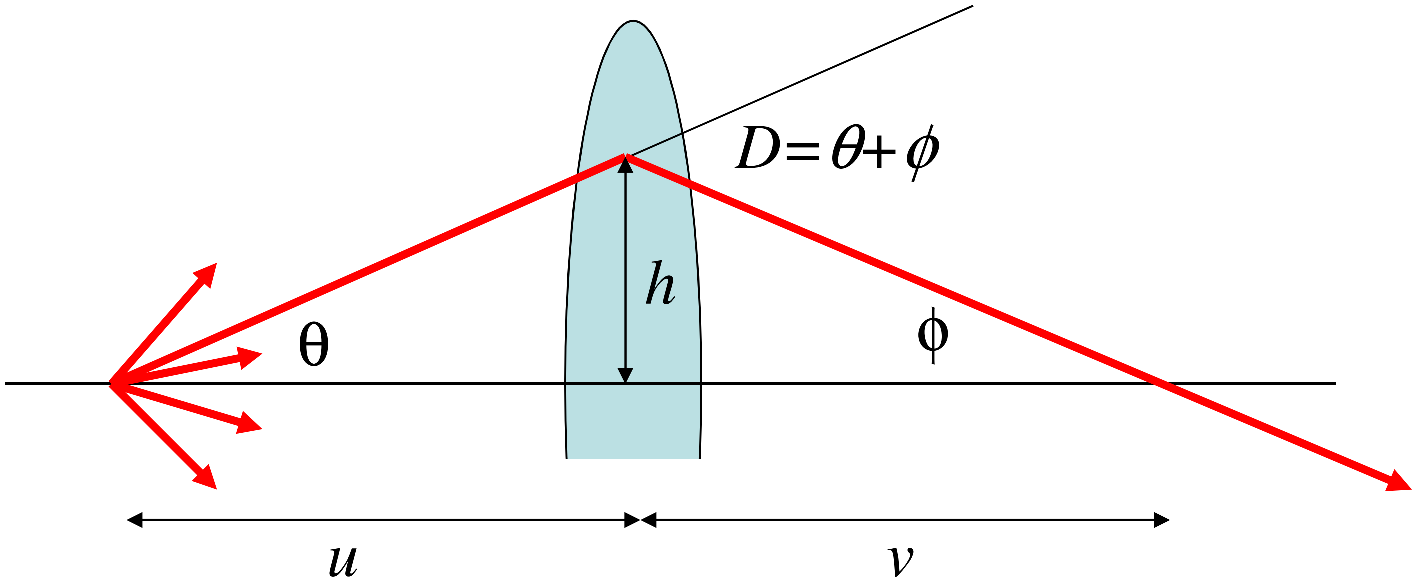
The lens formula (2)



A lens is formed by a pair of curved surfaces. The angle of the equivalent prism is the angle between the surface tangents, which equals the sum of α and β.

For spherical surfaces $\alpha \approx h/r_1$ and $\beta \approx h/r_2$ where r_1 and r_2 are the radii of the two spheres

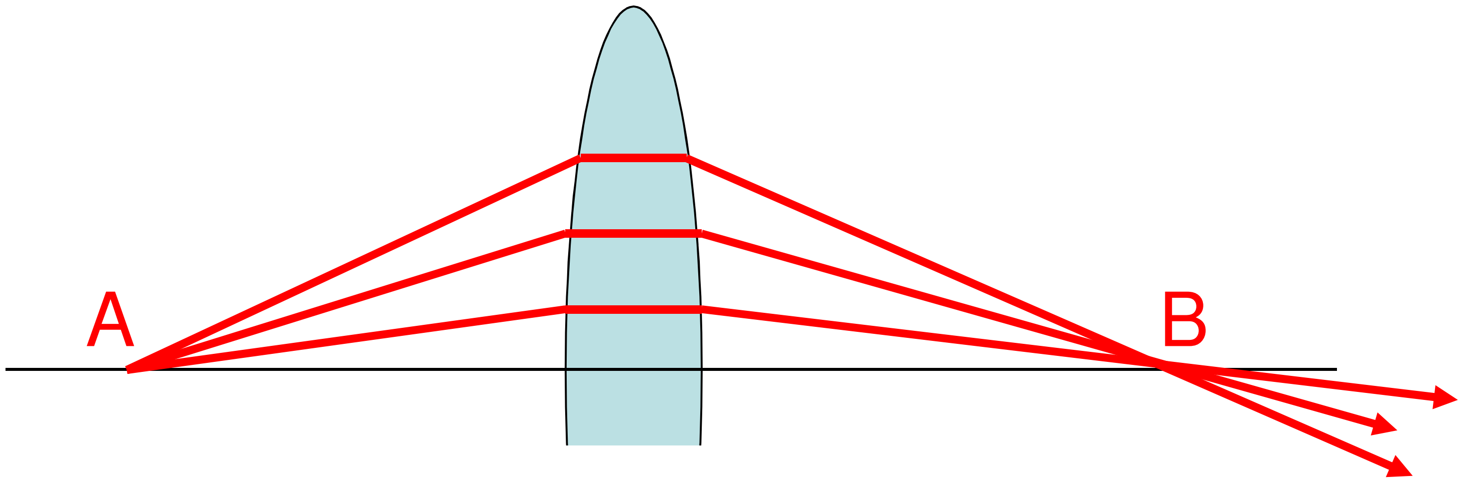
The lens formula (3)



$$h/u + h/v = (n-1) \times (h/r_1 + h/r_2)$$

$$1/u + 1/v = (n-1) \times (1/r_1 + 1/r_2) = 1/f$$

A lens (Fermat)



•Light can take several different paths from A to B

•All paths must be minimum time, so all must take the same time! Lens must be shaped so that extra length in air cancels shorter length in glass

Special cases

$$1/u + 1/v = 1/f = (n\text{-}1) \times (1/r_1 + 1/r_2)$$

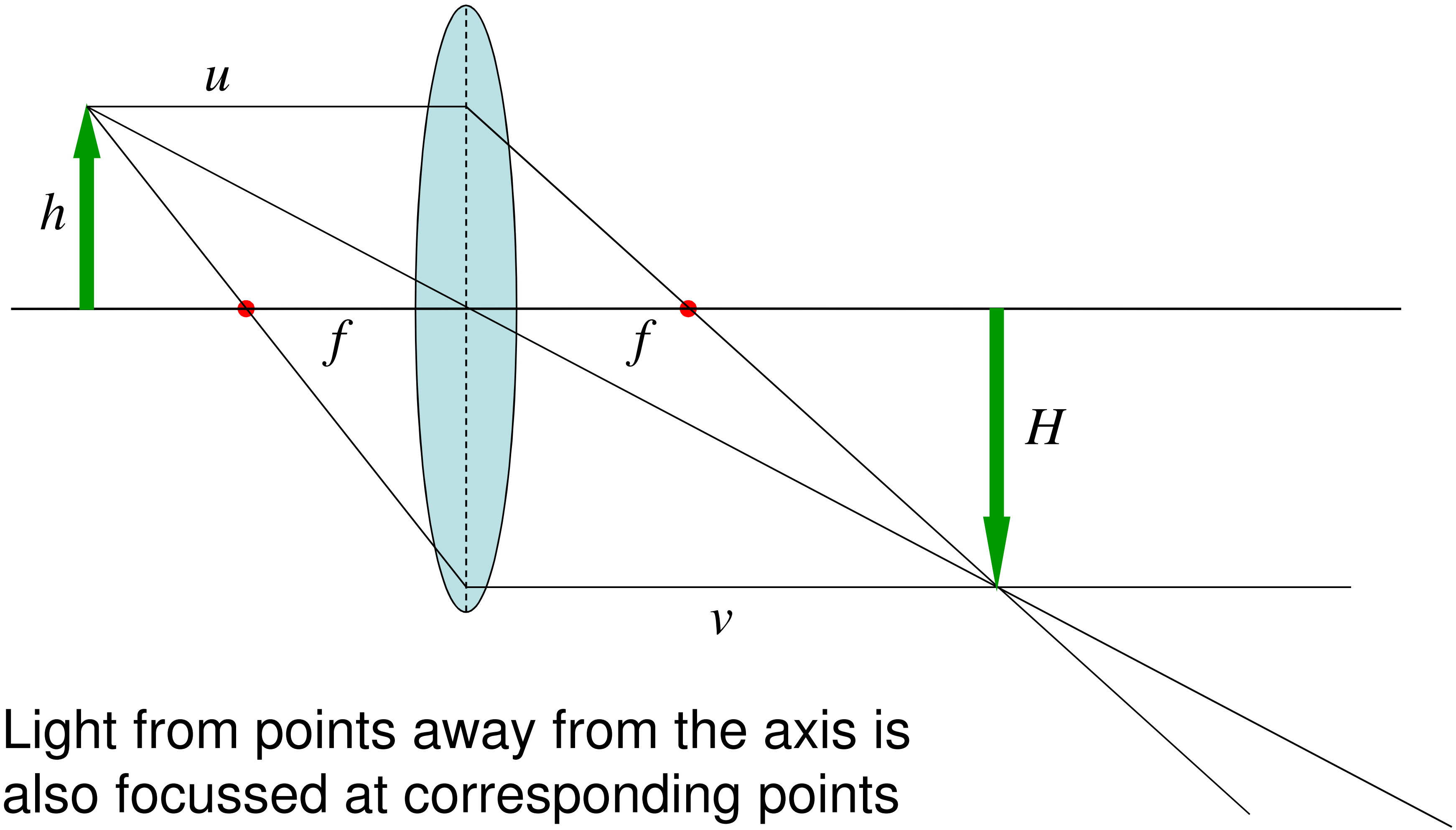
a) When the source is a long way away ($u \rightarrow \infty$) the light rays are parallel to the axis and are focussed onto the axis at a distance f, the focal length.

b) When the source is one focal length ($u = f$) away from the lens the light rays are focussed at infinity, forming a parallel beam (reciprocity!).

Newtonian form

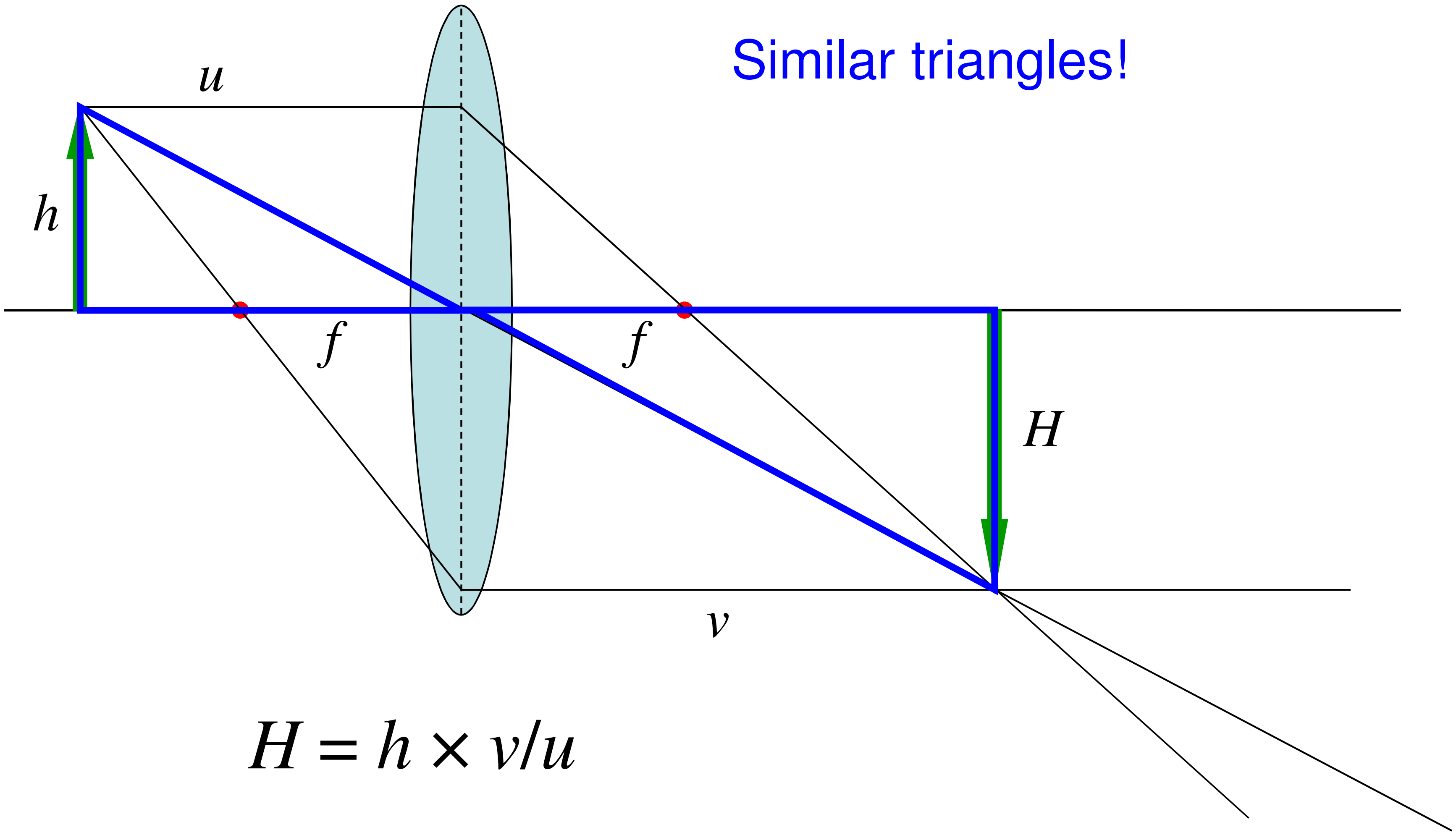
- The lens equation used above is in Gaussian form, in which all distances are measured from the lens centre
- Newton chose to measure all distances from the relevant focal point so that $u=f+x_o$ and $v=f+x_i$
- Solving the lens equation in these variables gives the Newtonian form $x_o x_i=f^2$
- Can be simpler to use in some cases

Extended objects



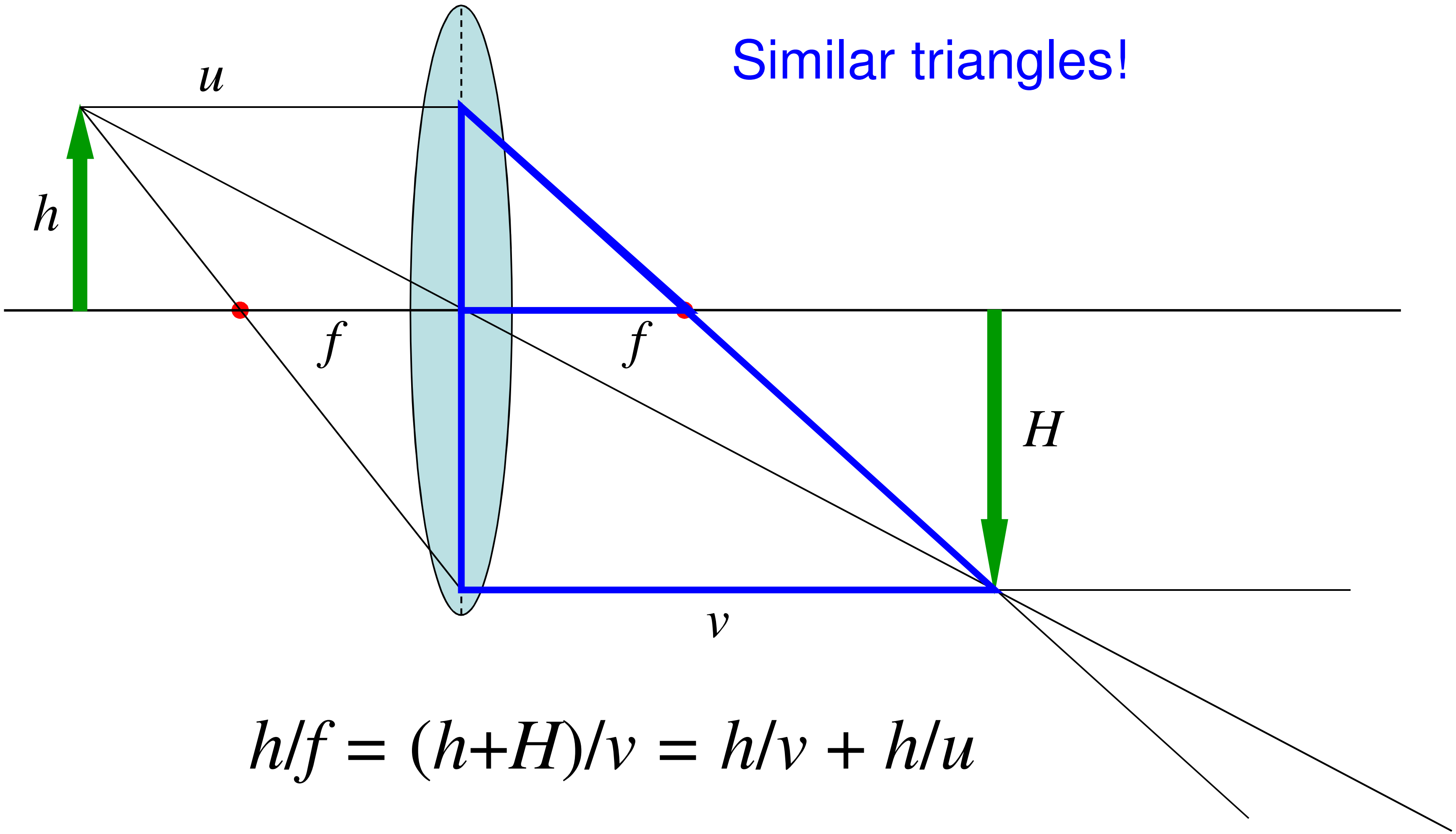
Light from points away from the axis is also focussed at corresponding points

Extended objects



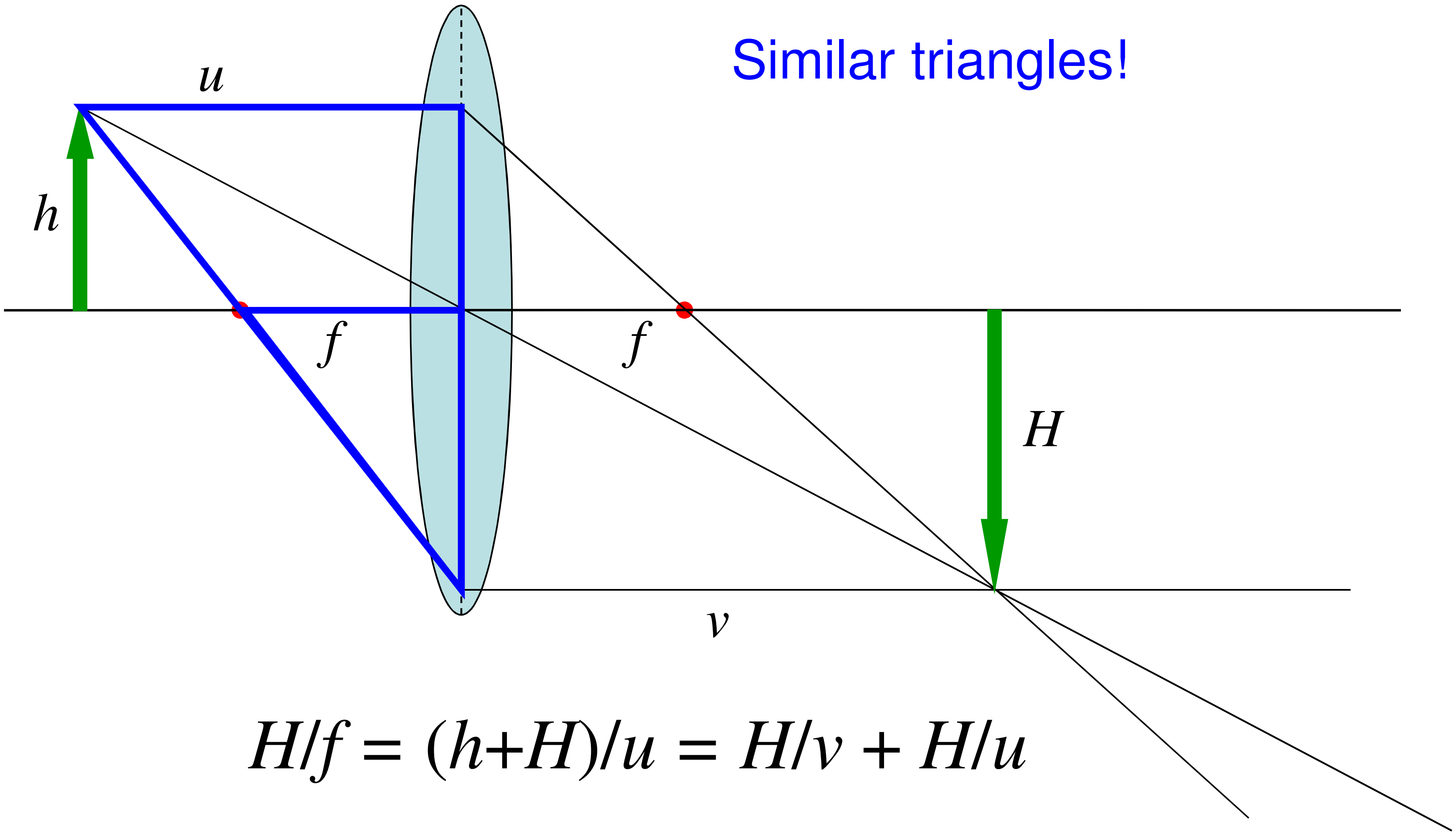
$$H = h \times v/u$$

Extended objects



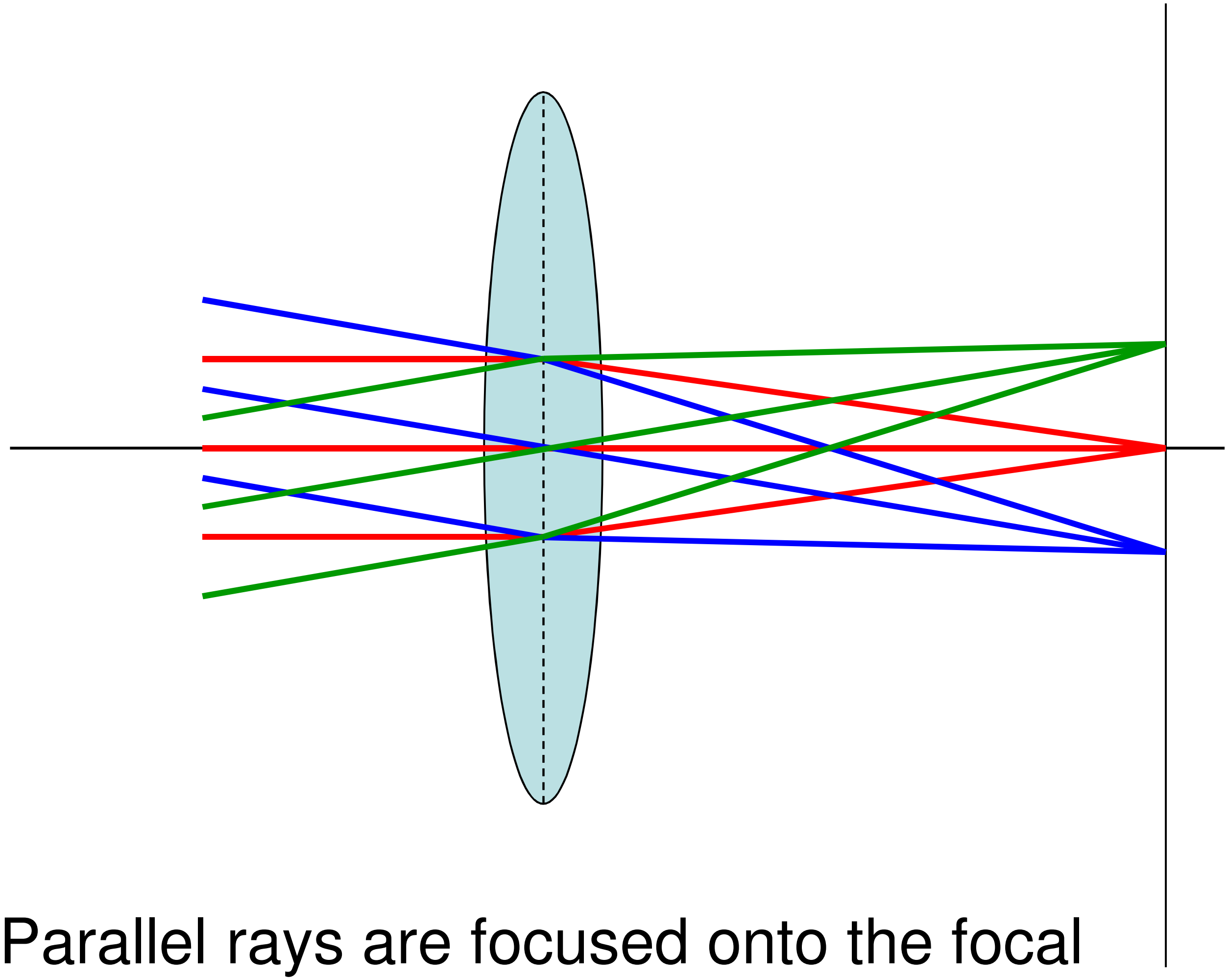
$$h/f = (h+H)/v = h/v + h/u$$

Extended objects



$$H/f = (h+H)/u = H/v + H/u$$

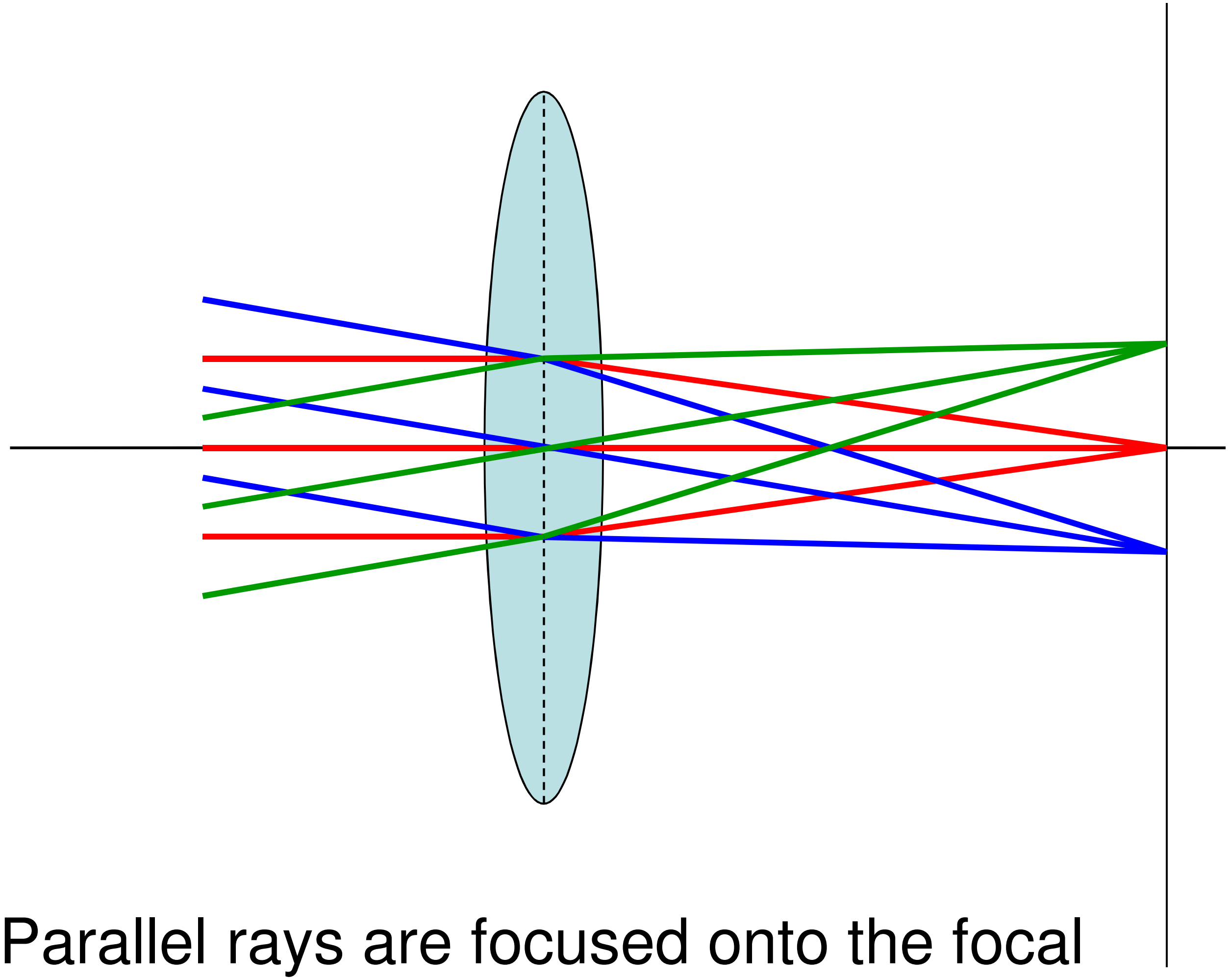
Focal plane



The lens converts angles to positions!

Parallel rays are focused onto the focal plane: in the limit $u \rightarrow \infty$ then $v \rightarrow f$

Landscape camera



Place film or a CCD detector in the focal plane. If the object is not at infinity then must move lens away from detector or decrease its focal length

Parallel rays are focused onto the focal plane: in the limit $u \to \infty$ then $v \to f$

Real images

- A converging lens will form a *real image* of an object on the opposite side of the lens, as long as the object is placed at least one focal length away.
- A real image can be directly detected using, for example, photographic film, a CCD chip, or just a piece of paper
- A real image can also be detected indirectly using a suitable optical system, such as a camera or a human eye, which comprises some lenses and a direct detector, such as film or the retina.

Magnification of real image

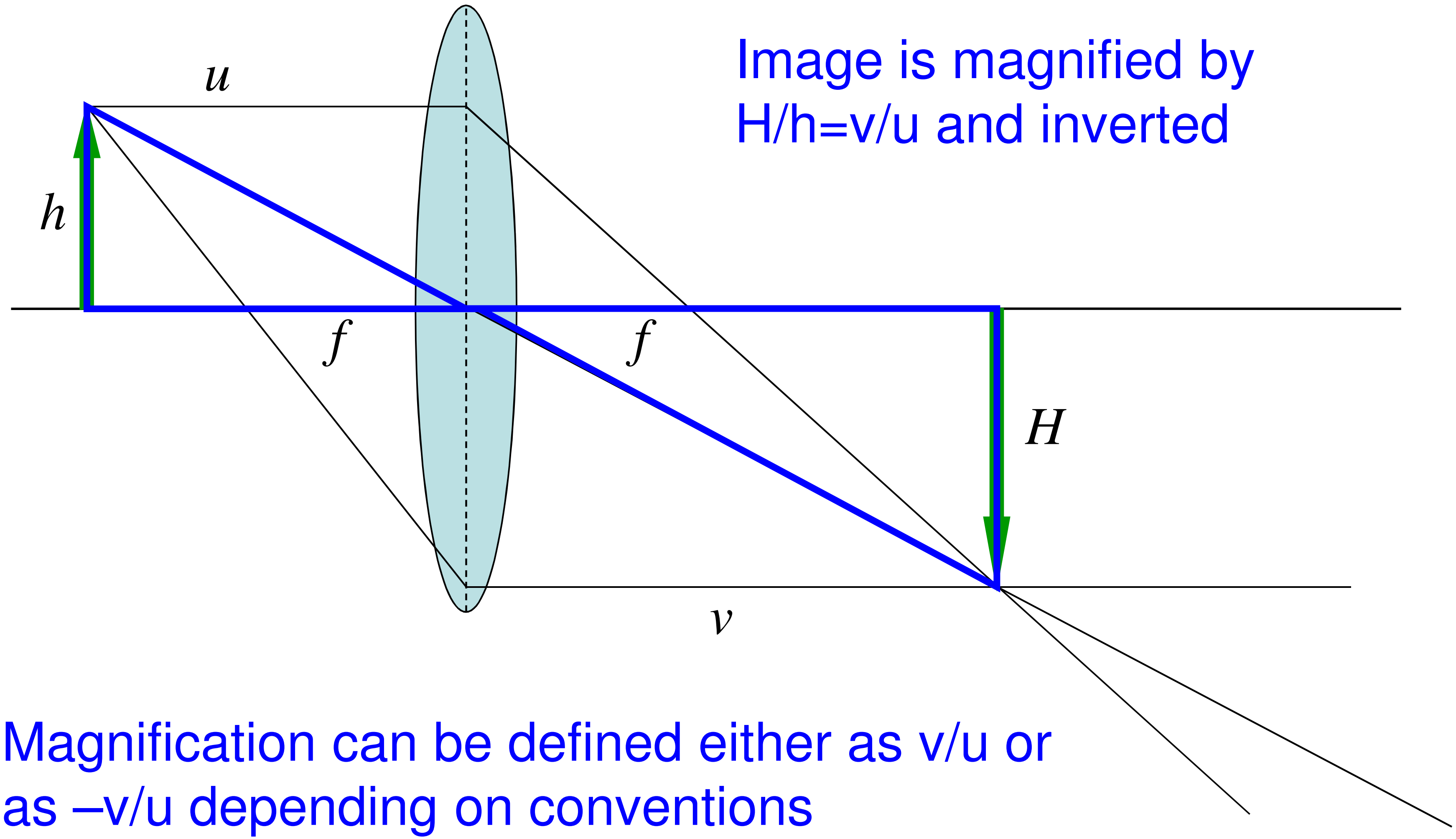
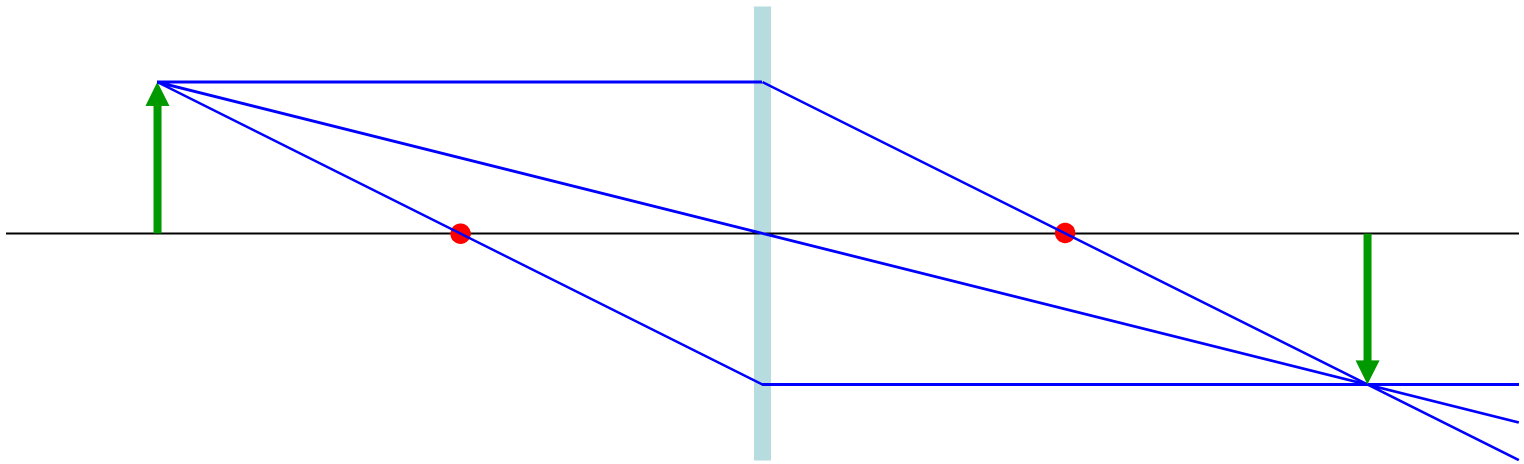


Image is magnified by H/h=v/u and inverted

Magnification can be defined either as v/u or as –v/u depending on conventions

Inverting lens

- A special case occurs when u=2f
- Solving 1/u+1/v=1/f gives v=2f
- Real image is inverted but same size as object



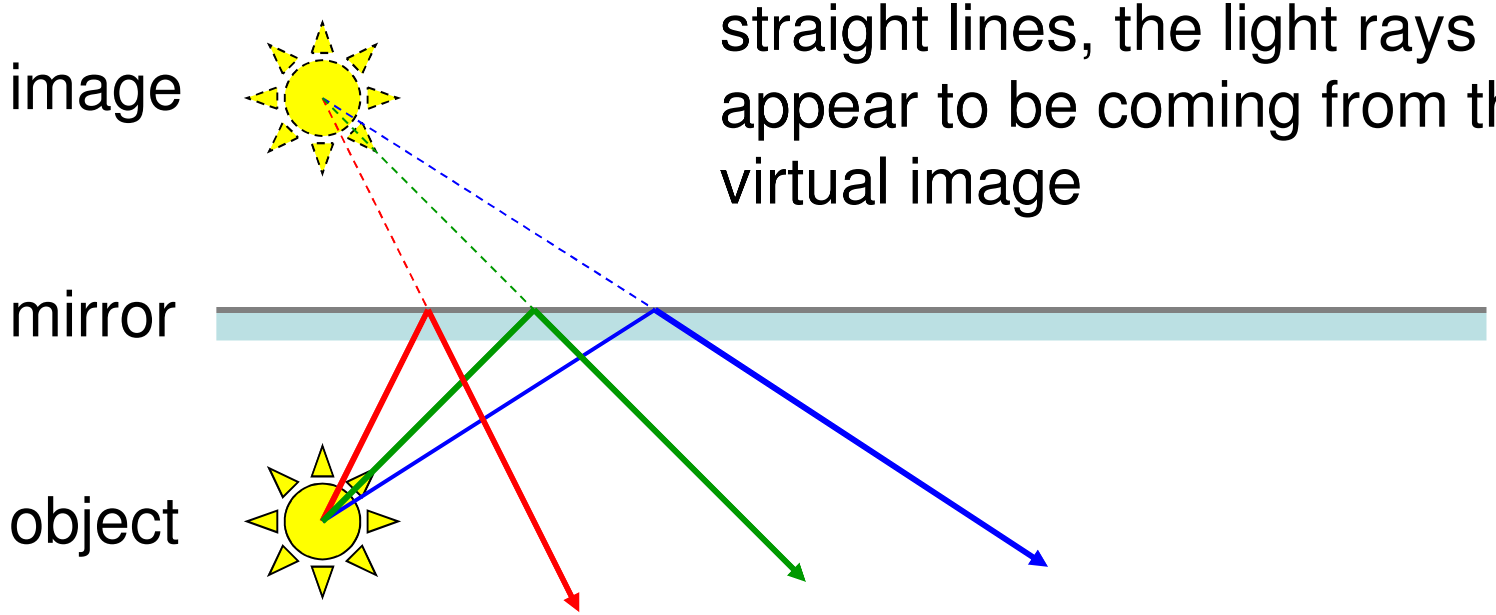
- Minimum distance between object and image

Virtual images

- A diverging lens will form a *virtual image* of an object on the *same* side of the lens.
- A virtual image cannot be directly detected using, for example, photographic film
- A virtual image *can* be detected indirectly using an optical system, such as a camera or a human eye.
- Virtual images are very familiar to us: the image in a plane mirror is a virtual image.

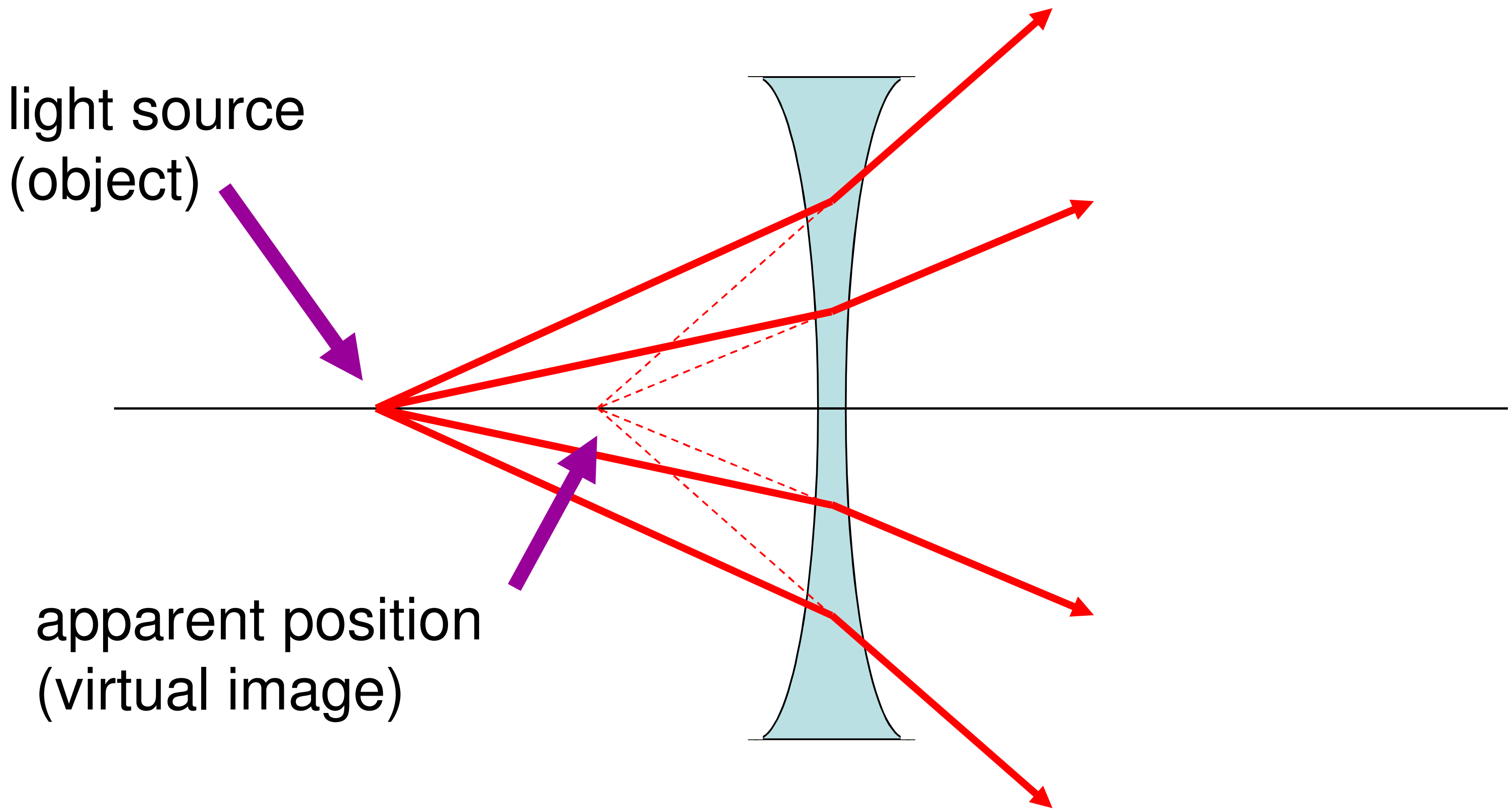
Virtual image in a mirror

Since light normally travels in straight lines, the light rays appear to be coming from the virtual image



To optical apparatus (eye)

Virtual image with a lens



Virtual images of extended objects are scaled down by v/u and upright (draw a ray diagram to check)

Lens summary

Object	Image type	Location	Orientation	Size
Converging Lens ($f>0$)				
$\infty>u>2f$	Real	$f<v<2f$	Inverted	Reduced
$u=2f$	Real	$v=2f$	Inverted	Same size
$2f>u>f$	Real	$2f<v<\infty$	Inverted	Magnified
$u=f$	Beam	$\pm\infty$		
$u<f$	Virtual		Erect	Magnified
Diverging lens ($f<0$)				
Anywhere	Virtual	$f<v<0$	Erect	Reduced

Lens formula (4)

$$1/u + 1/v = 1/f = (n-1) \times (1/r_1 + 1/r_2)$$

• The lens (makers) formula can be generalised to arbitrary lenses and to real and virtual images as long as an appropriate sign convention is used.

• For simple systems use the *real is positive* convention. Distances to real objects and images are positive and to virtual objects and images are negative. Radii of surfaces are positive if they cause deviations towards the axis and negative if they cause deviations away from the axis.

Lens as two surfaces

- The treatment of a lens given previously treats both surfaces simultaneously

- An alternative approach is to treat the two surfaces separately, treating a thin convex lens as two thin *planoconvex* lenses

- Systems of two lenses are non-examinable, but case of two thin lenses at same point is simple!

Lens power approach

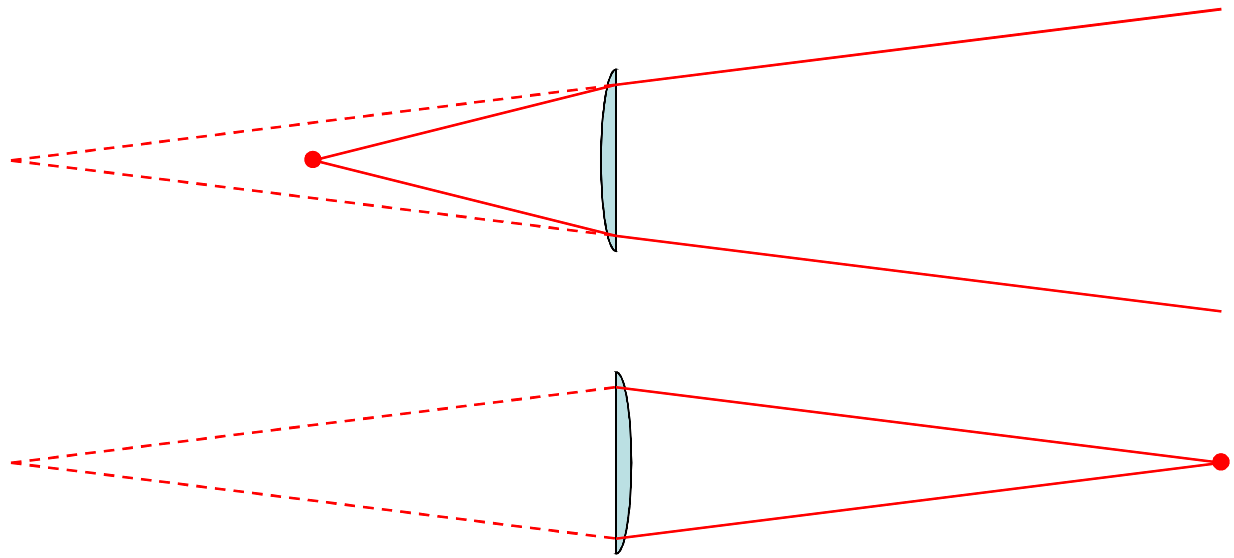
- Start from the formula for a planoconvex lens of radius R and refractive index n:
  $1/u+1/v=(n-1)/R=1/f$

- Thin lenses in contact combine by adding their *powers*, which are just the reciprocals of the focal lengths: $1/f=1/f_1+1/f_2$

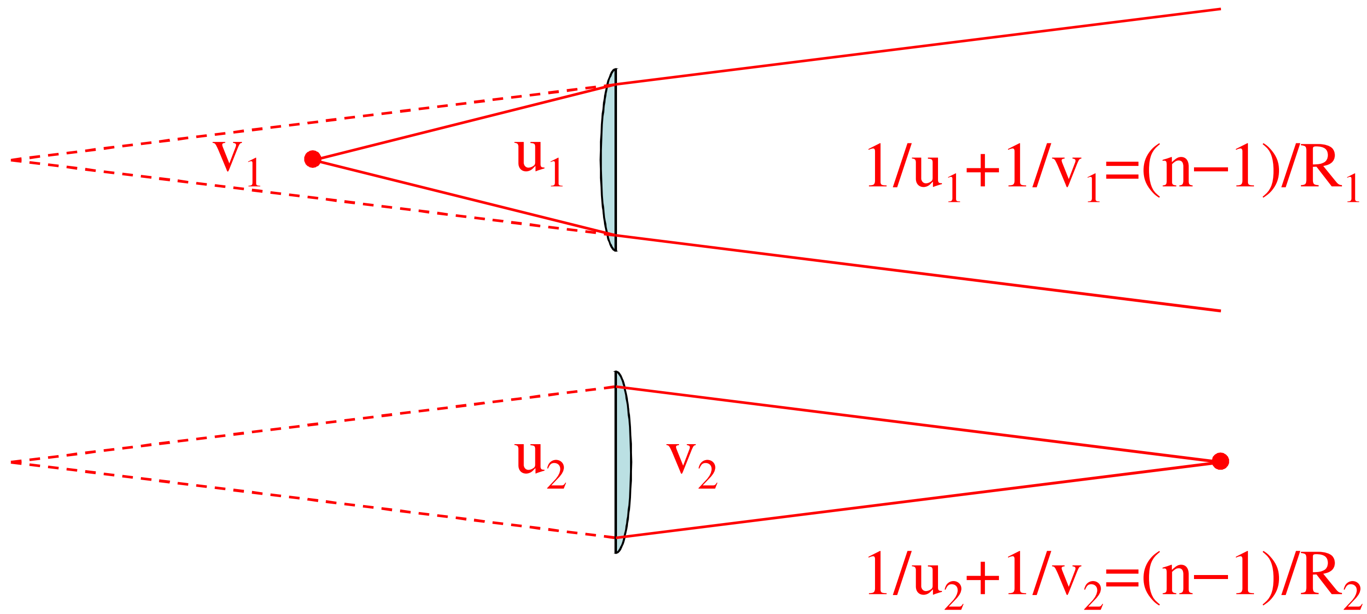
- Thus obtain $1/u+1/v=(n-1)[1/R_1+1/R_2]$

Direct approach (1)

- Where does this rule come from? Assume that the first surface creates a *virtual image* which acts as an object for the second surface



Direct approach (2)

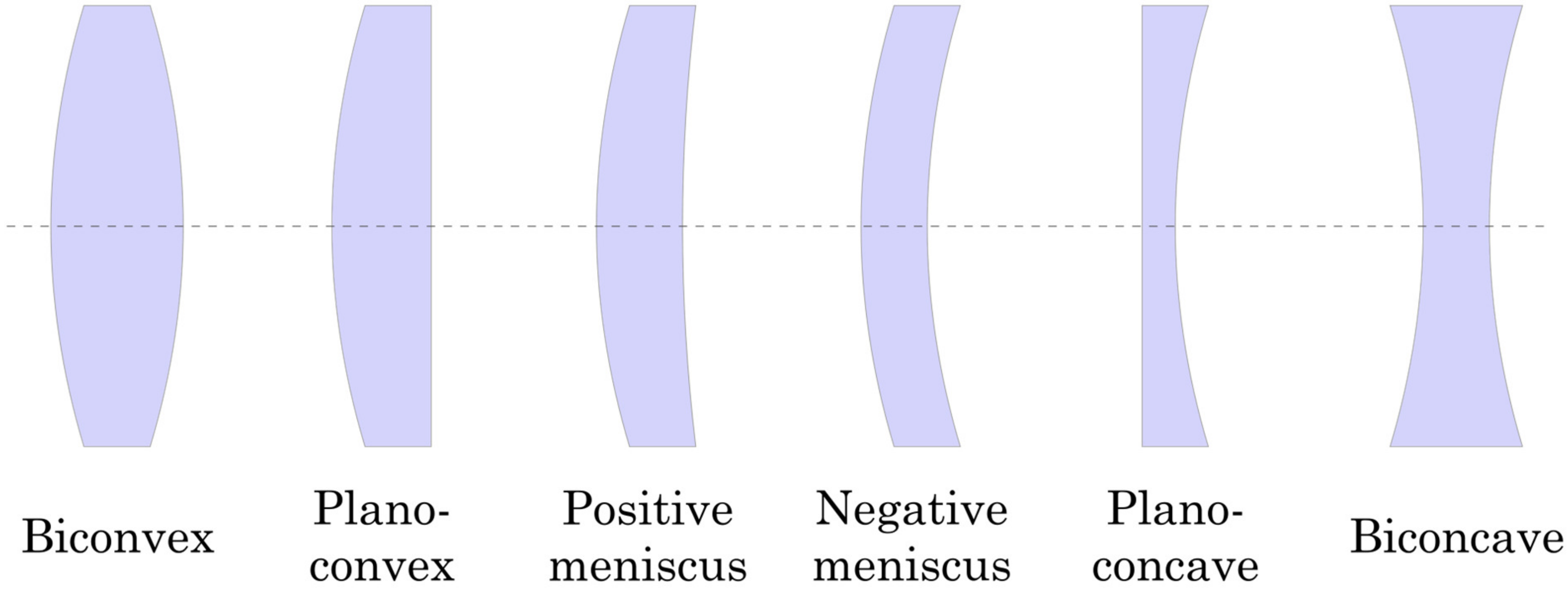


- Note that $u_1=u$, $v_1=-u_2$ and $v_2=v$, so adding equations gives desired result!

Sign conventions

- Treatments using this approach often use a *geometric* sign convention where the object is at a negative distance from the lens.
- The curvature of a spherical surface is positive if the centre lies at a positive distance from the surface and negative if the other way round.
- For a biconvex lens the first surface is positive and the second is negative
- Formula is $1/u+1/v=(n-1)[1/R_1-1/R_2]$

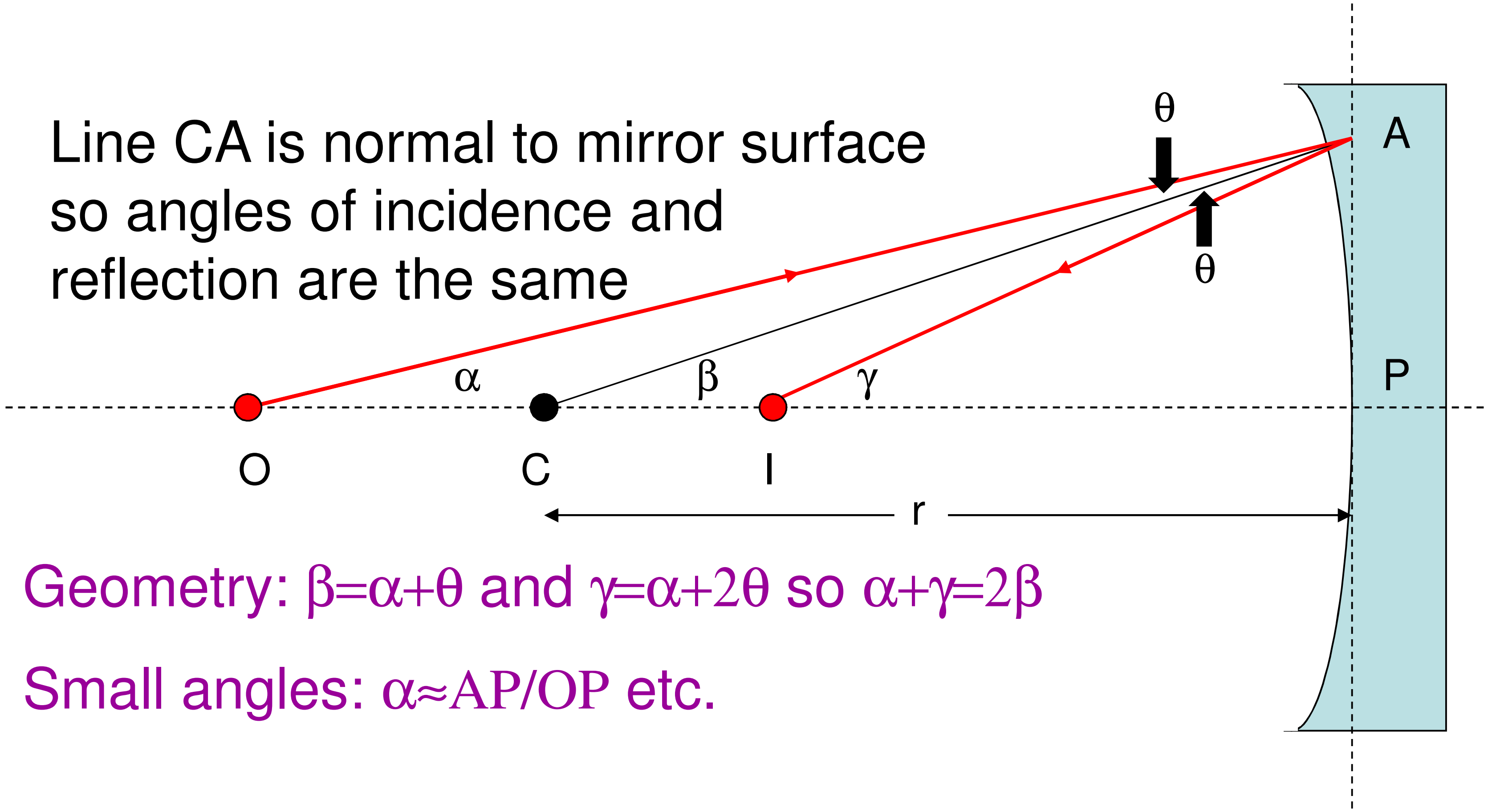
Lens types



Converging lenses Diverging lenses

The mirror formula

Line CA is normal to mirror surface so angles of incidence and reflection are the same



Geometry: $\beta = \alpha + \theta$ and $\gamma = \alpha + 2\theta$ so $\alpha + \gamma = 2\beta$

Small angles: $\alpha \approx AP/OP$ etc.

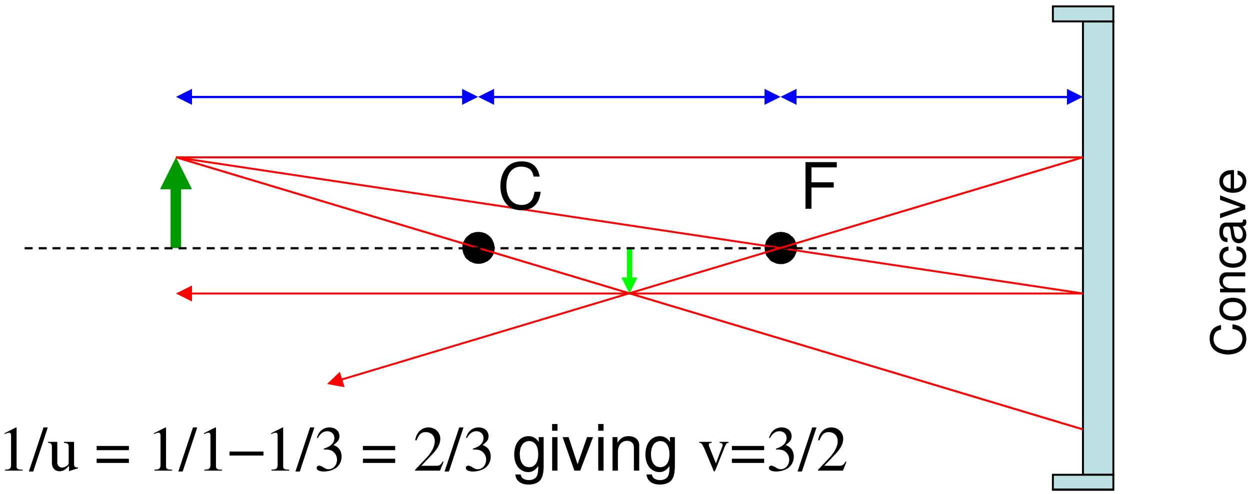
$$1/u + 1/v = 2/r = 1/f$$

Mirrors and images (1)

- A spherical mirror will form a point image of a point object under paraxial approximations
- The mirror formula can be generalised to arbitrary mirrors with a sign convention.
- Concave mirrors normally create real images *in front* of the mirror and have positive radii
- Convex mirrors create imaginary images *behind* the mirror and have negative radii.

Mirrors and images (2)

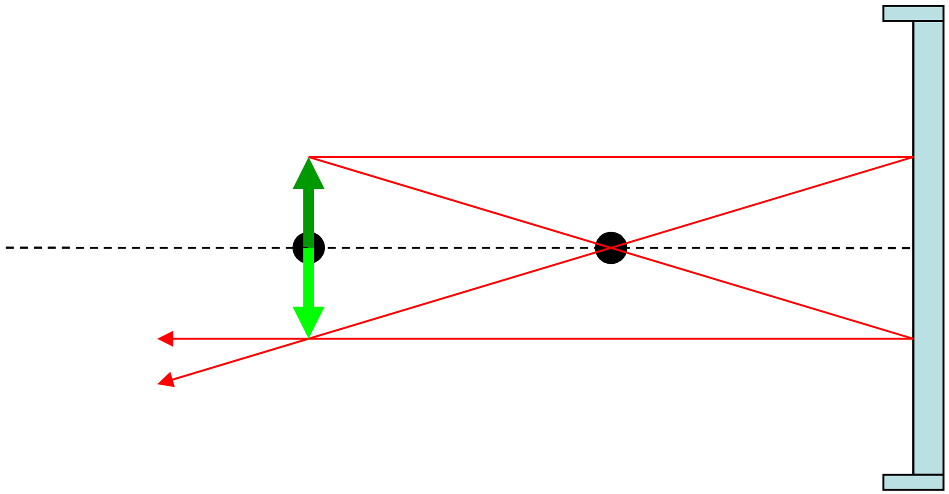
- A ray through the centre of curvature is reflected in the same direction. A ray through the focus is reflected parallel to the axis and *vice versa*.



$1/v = 1/f - 1/u = 1/1 - 1/3 = 2/3$ giving $v = 3/2$

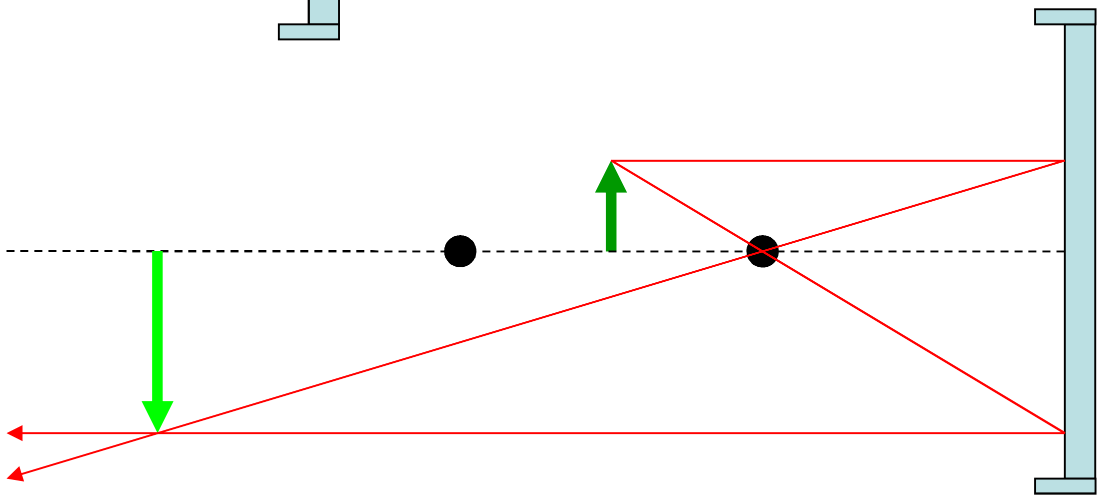
Image real and inverted and scaled by $v/u = 1/2$

Mirrors and images (3)

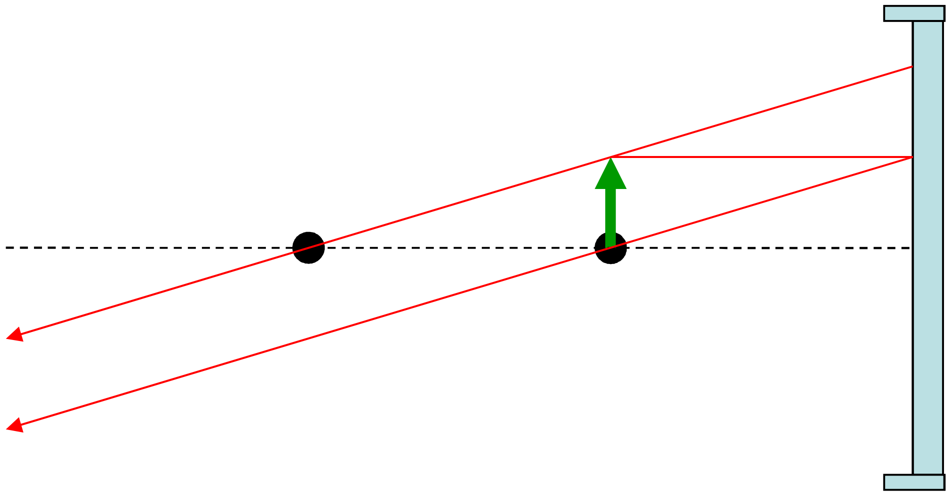


Object at C gives a real inverted unmagnified image at C

Object between C and F gives a real inverted magnified image beyond C

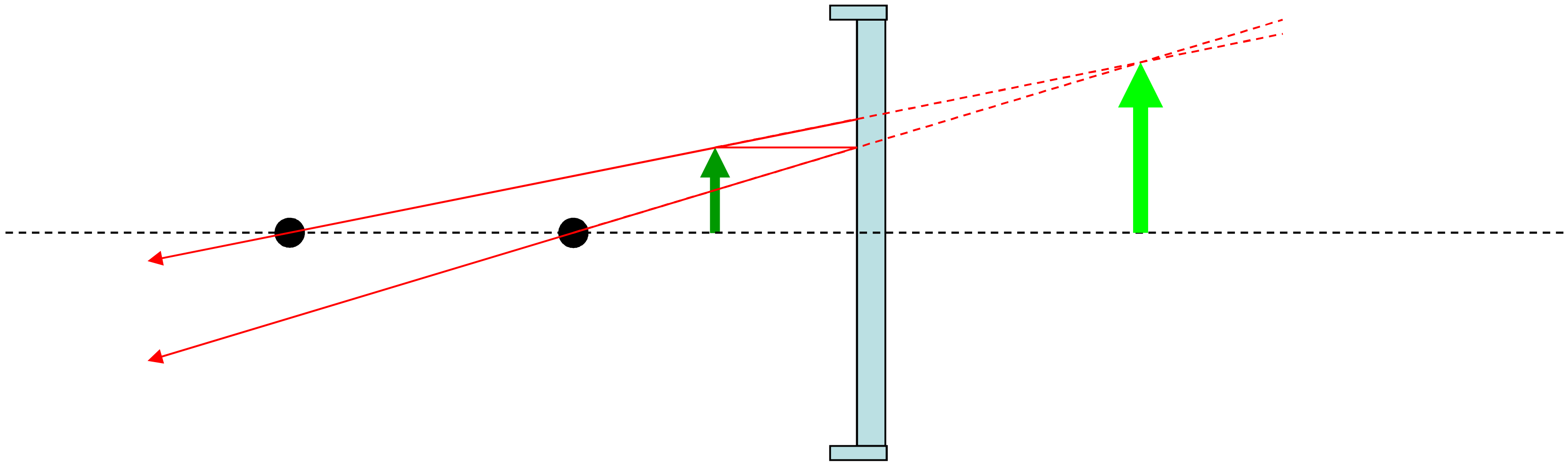


Mirrors and images (4)



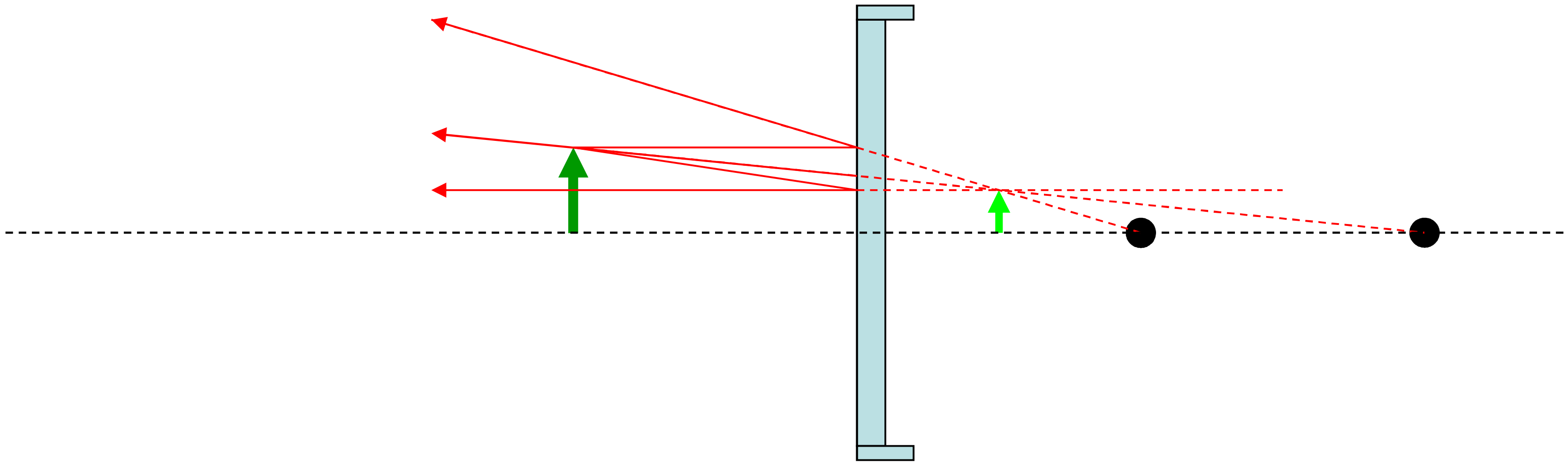
Object at F gives no image but creates a parallel beam of rays with different heights appearing at different angles

Mirrors and images (5)



Object closer than F gives an enlarged upright *virtual* image behind the mirror. This is how shaving/makeup mirrors work. Magnification is greatest when the object is near F.

Mirrors and images (6)



With a convex mirror the image is always virtual and located behind the mirror and is always *smaller* than the object

Mirror sign conventions

- *Sign conventions in treatments of mirrors are almost hopelessly confused*
- Real is positive *vs* Geometric
- Measuring geometric distances left to right *or* along the light direction
- Negating magnification *or* not

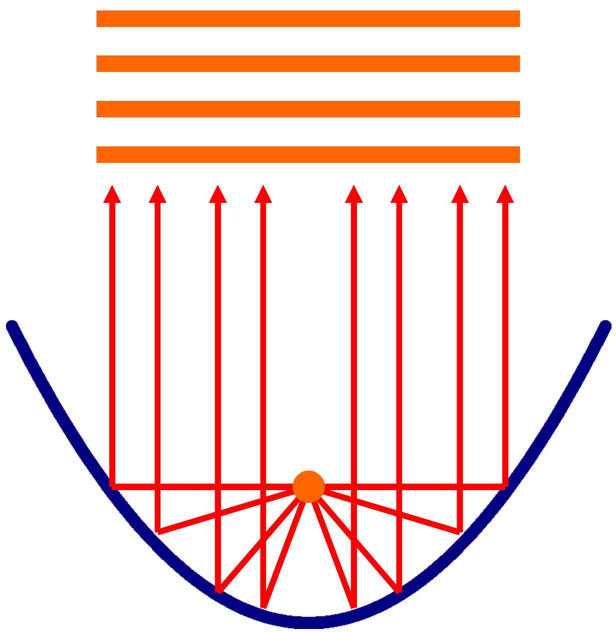
- Usually just have to work it out

Parabolic Mirrors (1)

- From the mirror formula we see that if a light source is placed at the focal length (r/2) from a spherical mirror then an image will be formed at infinity, indicating that a parallel beam of light is produced
- But it is well known that a *parabolic* mirror is necessary to create a parallel beam! What's going on?

Parabolic Mirrors (2)

- We can deduce the shape needed from Fermat's principle or from straight wavefronts

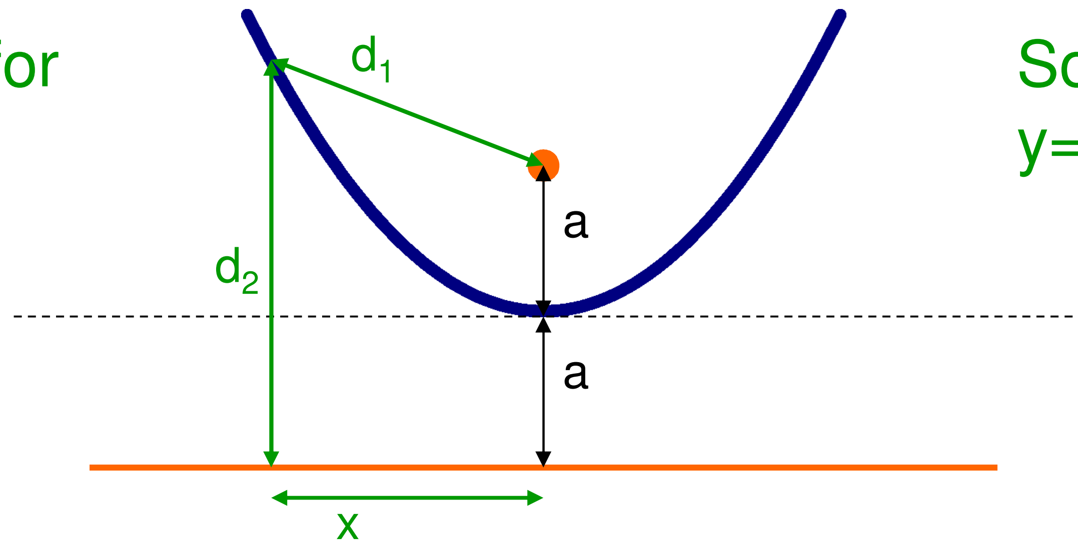


Place a source as indicated at the "focus" of the mirror. This will produce plane wavefronts if the red lines all have the same length

Parabolic Mirrors (3)

- Geometric definition of a parabola: the locus of all points equidistant from a line and a point at a distance 2a from the line (the focus)

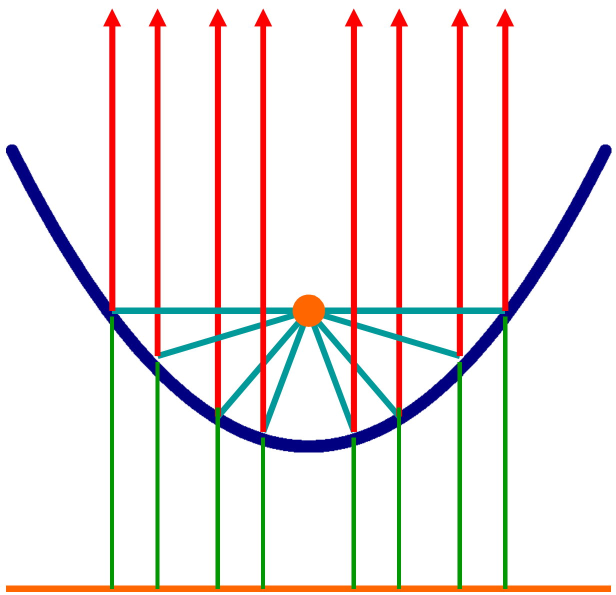
Solve for $d_1=d_2$



Solution is $y=x^2/4a$

Parabolic Mirrors (4)

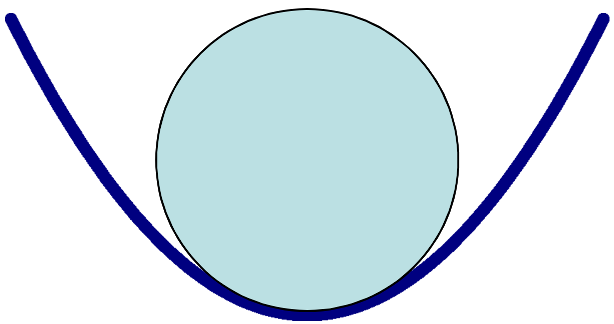
- We can immediately deduce that a parabolic mirror will produce a beam of light from a source placed at its focus



From the definition of a parabola the green lines all have the same length as the corresponding blue lines. Thus all the optical paths have the same total length!

Parabolic Mirrors (5)

- So what about spherical mirrors?



The bottom of a parabola looks pretty much like a circle!

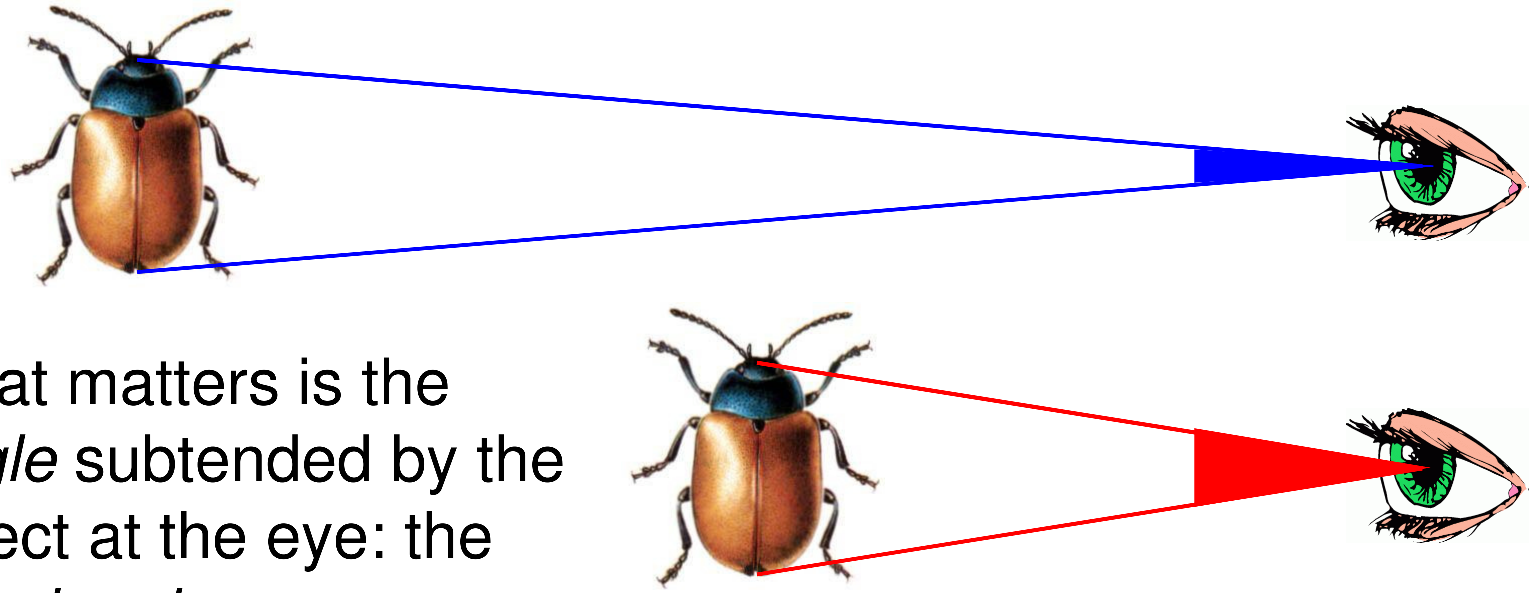
Consider a circle radius r centred at (0,r)

$$y = r - \sqrt{(r^2 - x^2)} \approx x^2/2r$$

A small portion of a spherical mirror radius r looks just like a parabolic mirror with a focus a=r/2. Thus for paraxial rays we can use spherical mirrors!

Angular size

- A small object looks larger if brought nearer to the eye



What matters is the *angle* subtended by the object at the eye: the *angular size*.

Near point

- A small object looks larger if brought nearer to the eye.
- The lens of the eye can vary its focal length so as to create a sharp *real image* on the retina for objects at different distances.
- The range of focal lengths of the eye is limited: it can focus on objects between *infinity* and the *near point* or *least distance of distinct vision* $D \approx 250$ mm.
- Far sighted people have a larger value of D and so cannot focus close up. Short sighted people cannot focus at infinity. These can be corrected with simple lenses ("glasses")

Magnifying glass (1)

- A small object looks larger if brought nearer to the eye.
- But if the object is brought closer than D then the eye cannot focus on it, which limits this approach

- A magnifying glass is a converging lens which can be placed in front of the eye enabling the eye to focus on objects much nearer than D and so see bigger images

Jeweller’s Loupes

Magnifying glass (2)

- The simplest way to think about a magnifying glass is that it *increases* the *power* of the eye's lens (decreases its focal length) so that it can focus on objects at a distance d nearer than D
- This increases the angular size of the object, and thus its apparent size, by the ratio D/d
- Equivalently the lens forms a *virtual image* of the object which the eye can focus on
- Usually explained by drawing ray diagrams and spotting similar triangles, but these can be confusing
- Use algebra instead!

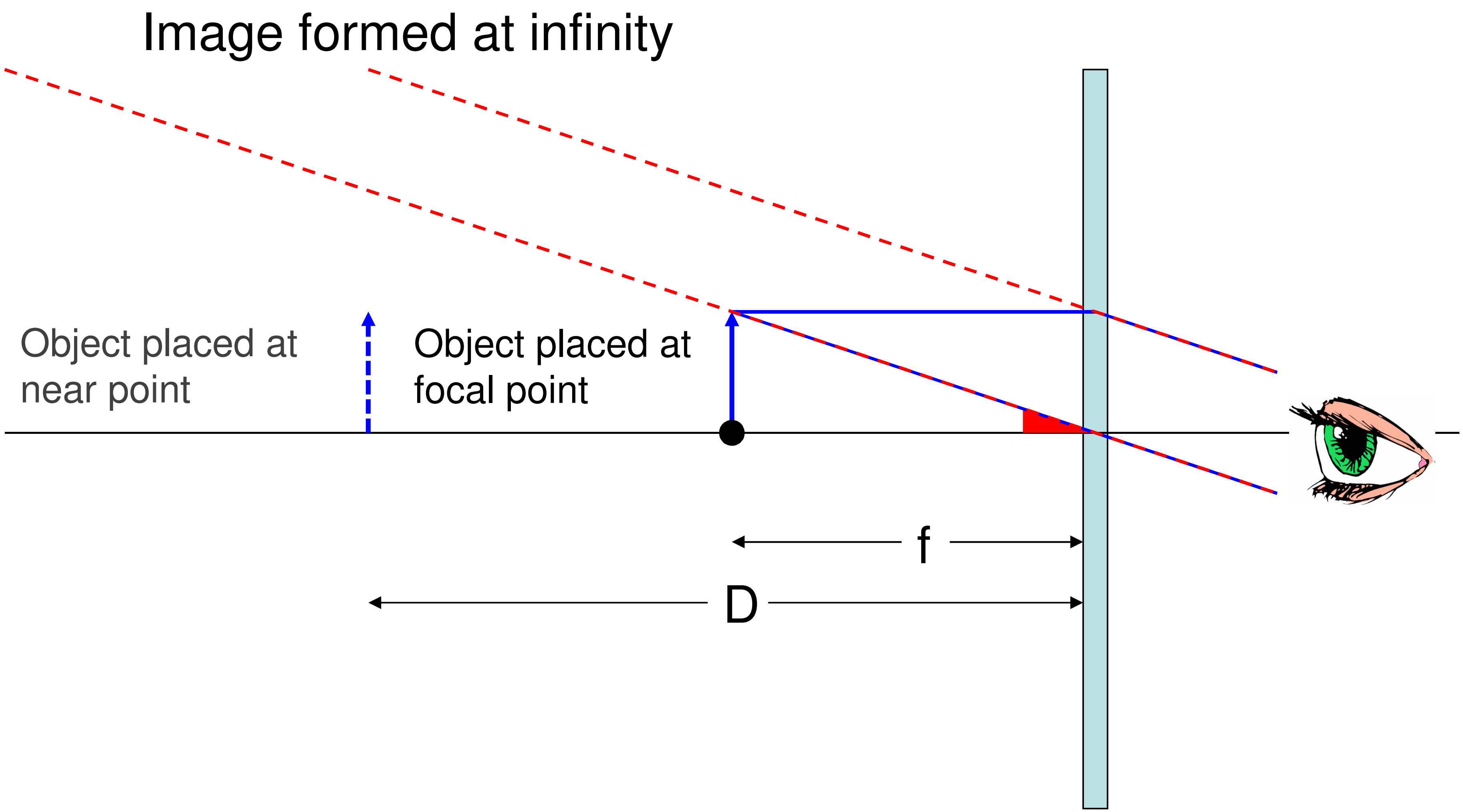
Magnifying glass (3)

- Assume the magnifying glass is a thin converging lens placed directly in front of the eye
- If the object is placed at a distance f from the lens then the virtual image will be formed at infinity, and it is easy for the eye to focus on this

- The achievable magnification is then given by D/f

- For example, a typical Loupe has f=2.5cm, and so will give a magnification of 10 when the object is placed 2.5cm away

Magnifying glass (4)



Magnifying glass (5)

- In fact you can get a slightly higher magnification by placing the object slightly closer than f
- This will create a virtual image nearer than infinity. As long as it is no closer than D the eye can focus on it.

- Limiting distance comes from solving $1/u-1/D=1/f$ to get $u=Df/(D+f)$

- Maximum magnification is then given by $D/u=D/f+1$

Magnifying glass (6)

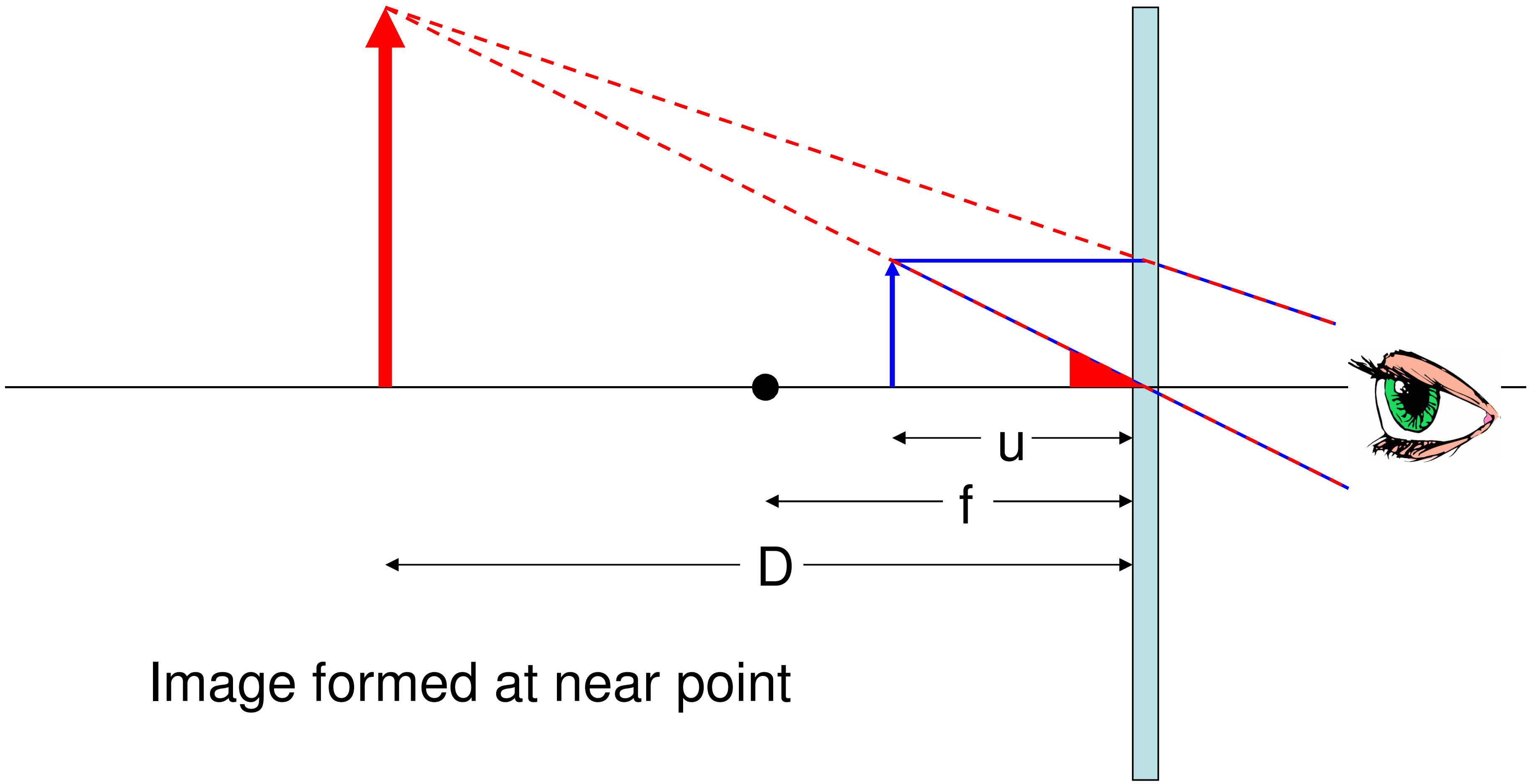


Image formed at near point

Magnifying glass (7)

- This is not actually how most people use magnifying glasses!
- Instead they place the magnifying glass close to the object and observe from a distance
- This is fairly simple to analyse in the case where the magnifying glass is placed one focal length away from the object
- Image is formed at infinity with angular size determined by the distance of the object from the lens
- Magnification is then given by D/f

Magnifying glass (8)

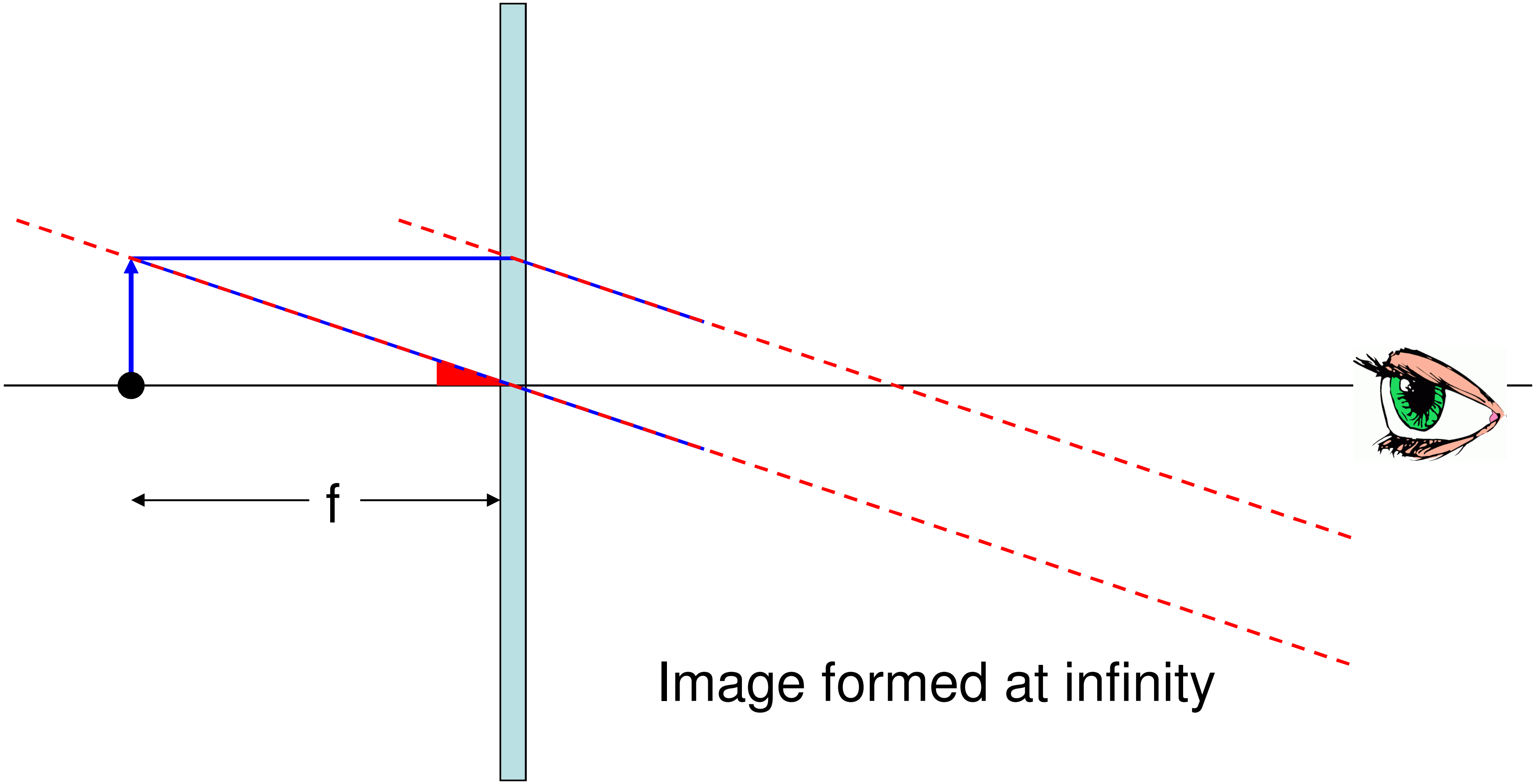
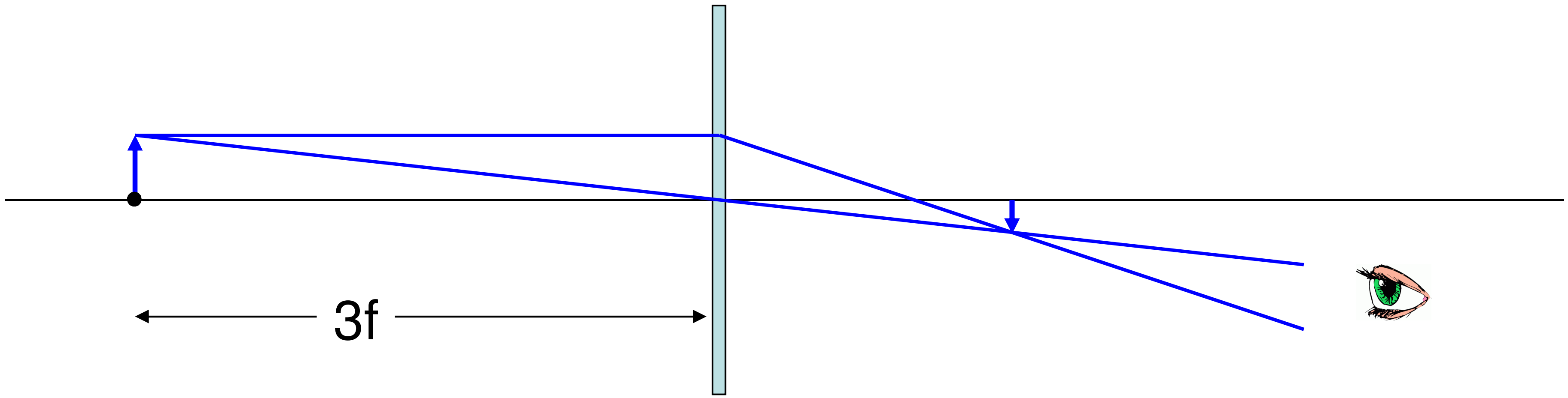


Image formed at infinity

Magnifying glass (9)

Real image formed between the lens and the eye



Seen as a reduced inverted image

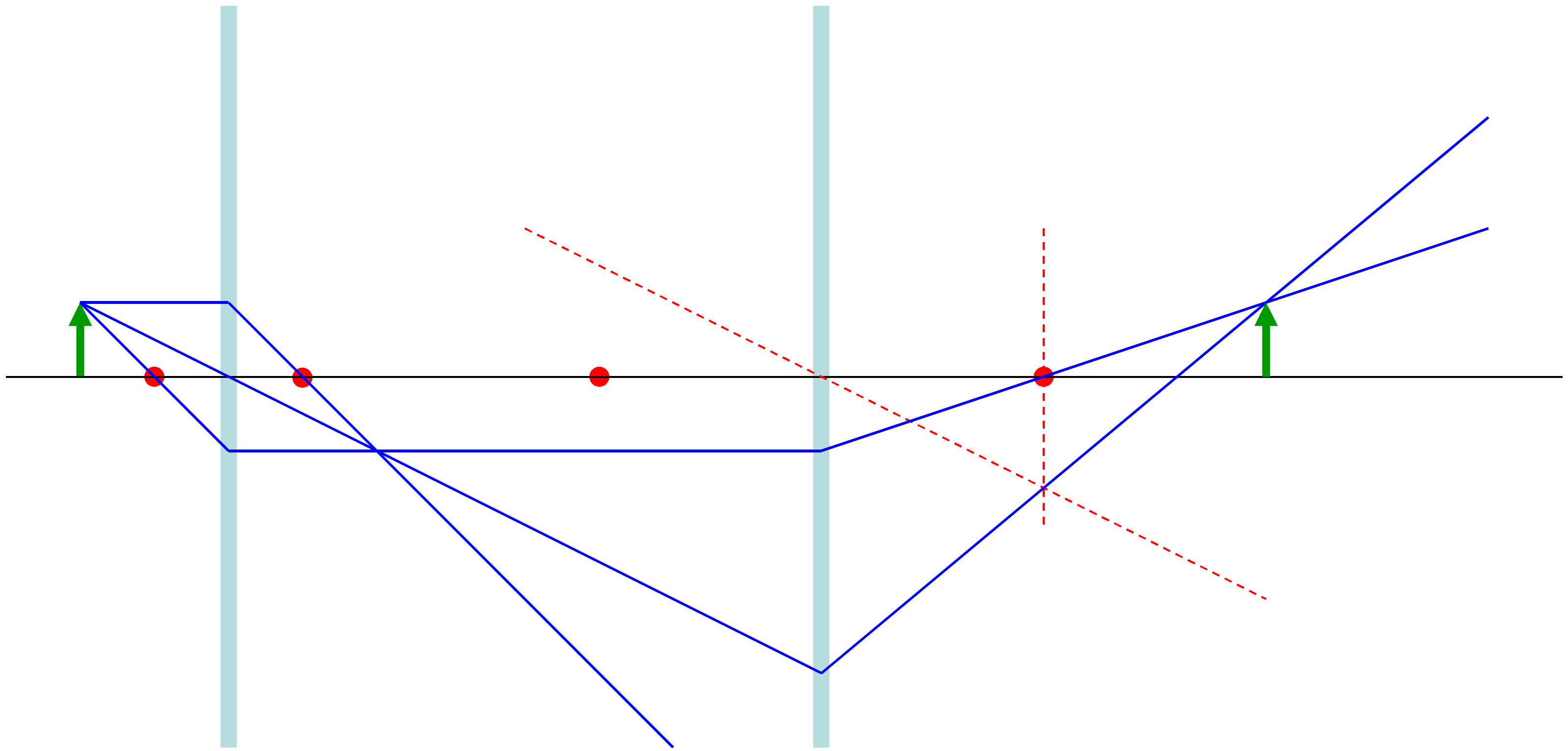
Optical instruments

- A single lens can be used to explain magnifying lenses and glasses/contact lenses but most useful optical instruments contain two or more lenses or mirrors.
- Details are largely non-examinable but we will study systems of two “lenses” in some detail and look in outline at more complex systems.
- Will continue to work with the thin lens and paraxial approximations. In reality multi-lens systems are highly vulnerable to the breakdown of these approximations giving rise to “aberrations” which we will only touch on.

Ray tracing with two lenses (1)

- We can calculate the paths of light rays in multi-lens systems by treating them as a series of singles lenses one after another
- The simple methods of ray tracing, however, become complicated as a ray which is "special" for one lens (parallel to the axis, or through the centre, or through the focus) will not be "special" for the next lens
- Key trick is to use "assistant rays" parallel to the ray of interest which pass through the centre of the second lens. The assistant ray will then meet the ray of interest in the focal plane of the second lens.

Ray tracing with two lenses (2)

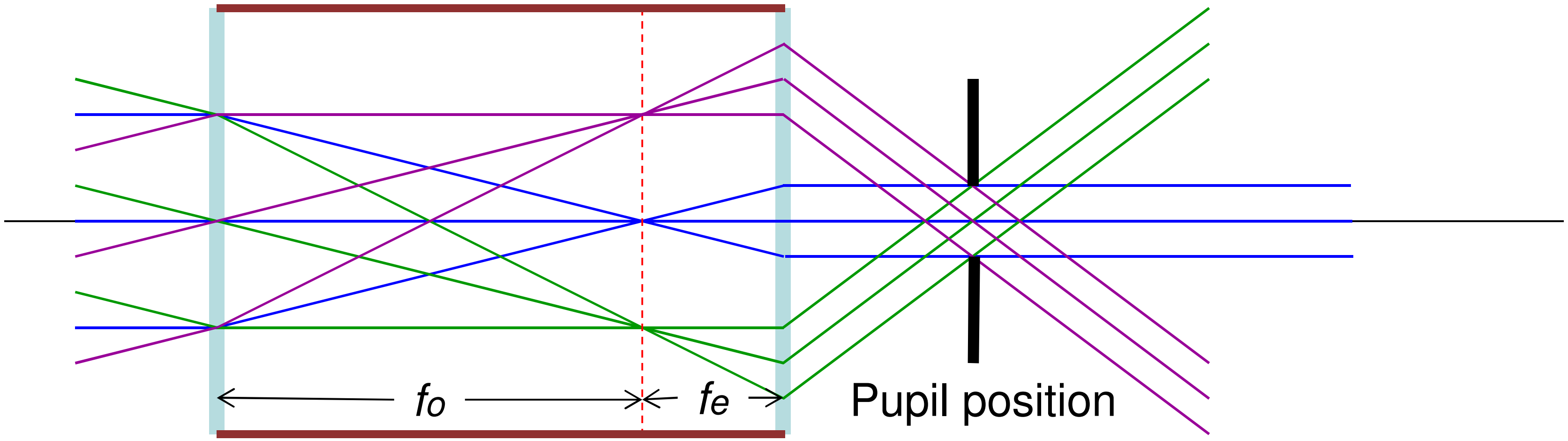


Can ray trace in complex systems by constructing “assistant rays” parallel to the ray of interest and passing through the centre of a lens

Astronomical telescope (1)

- The simple astronomical telescope is constructed out of two converging lenses and is used to observe an object which is far away, effectively at infinity.
- The first converging lens forms a real inverted image of the object. This could be observed directly with a detector such as a CCD chip, but in general we want to observe with an eye.
- The second converging lens forms a virtual image of this intermediate image at infinity so that the eye can easily focus on it.

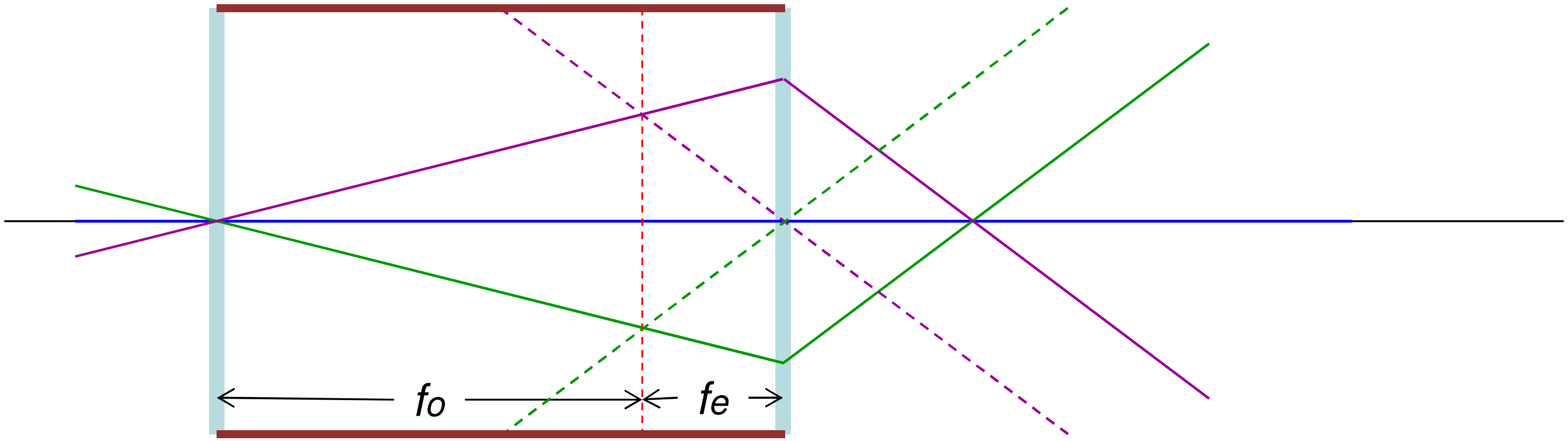
Astronomical telescope (2)



Two converging lenses (objective and eyepiece) are separated by the sum of their focal lengths f_o and f_e (red dashes mark the common focal plane)

Optimal pupil position is behind the eyepiece lens as indicated. In other positions the field of view is reduced

Astronomical telescope (3)

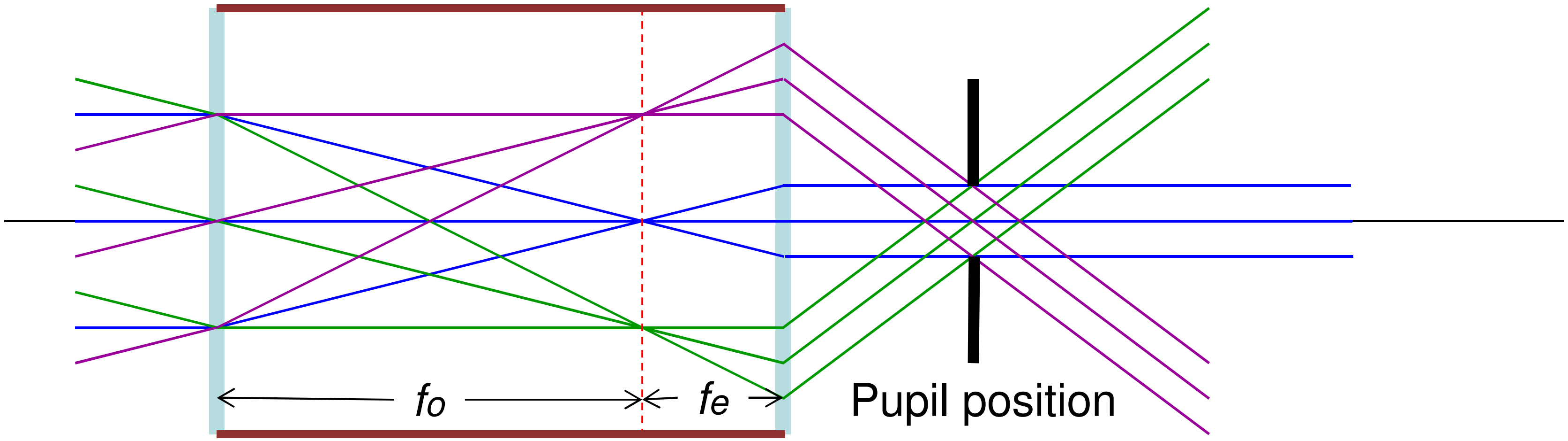


To find the angular magnification consider only the rays which pass through the centre of the first lens

Draw lines parallel to these rays which pass through the centre of the second lens

The image is inverted and magnified by a factor f_o/f_e (by triangles with focal plane and optical axis)

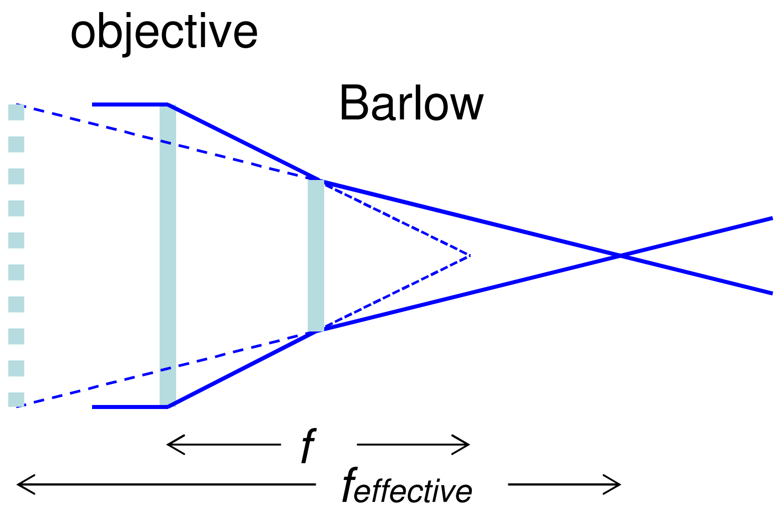
Graticules



Anything placed at the focal plane will also be easily observed by the eye. If a graticule (a grid of wires) or a reticle (cross hairs) is placed here then it will be clearly visible superimposed on the image of the distant object. Can be used for measurements or in telescopic sights.

Practicalities (1)

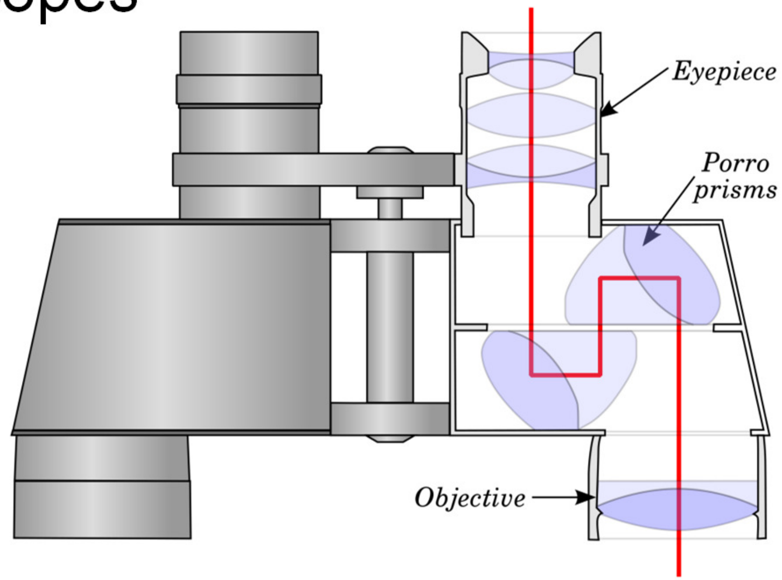
- To get the maximum magnification need to increase f_o or decrease f_e. A large value of f_o requires a very large telescope, while a small value of f_e requires a very powerful lens which will have serious aberrations. This can be partly overcome using a diverging *Barlow lens* which increases the effective value of f_o.



Practicalities (2)

- If the inversion of the image is a problem it can be overcome using an *inverting lens* (or more practically a pair of lenses) or by reflecting the image twice using mirrors or prisms set to give total internal reflection.
- Widely used in terrestrial telescopes and binoculars, but irrelevant in astronomical telescopes

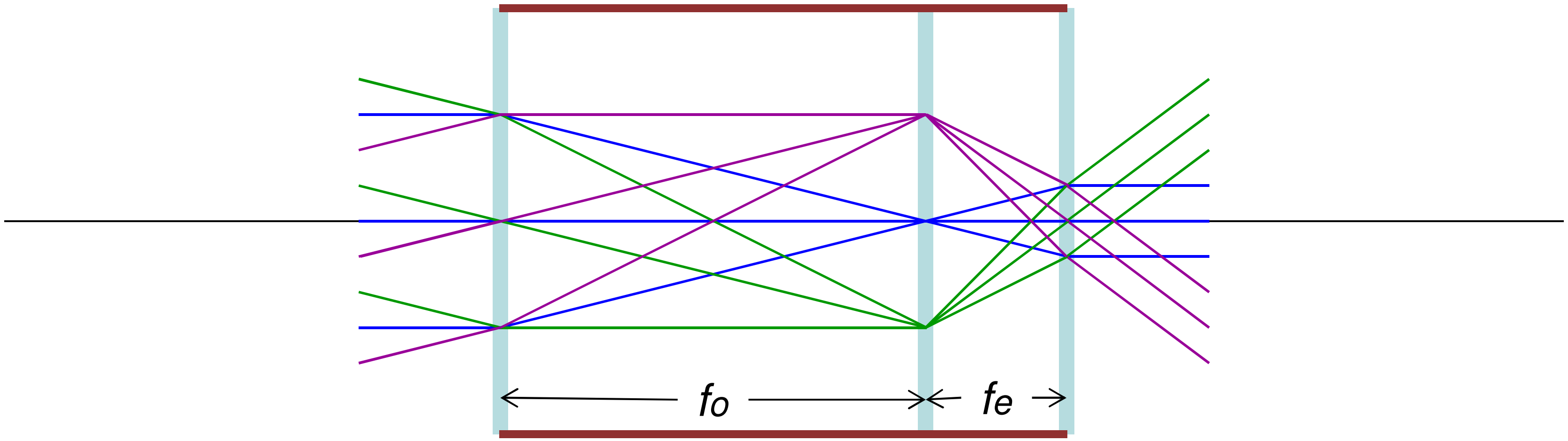
Inverting lenses will lengthen the telescope tube, but mirrors can shorten the tube by folding it up



Practicalities (3)

- The diameter of the "exit pupil" (the width of the bundle of parallel rays at the ideal pupil position) is given by the diameter of the objective lens divided by the magnification of the telescope.
- If the exit pupil is smaller than your actual pupil (7mm under ideal conditions) then the image will appear less bright than the object viewed directly, so high magnification telescopes require very large objective lenses making them very heavy.
- The *position* of the exit pupil can be moved using a *field lens* in the focal plane

Field lens (1)



A converging lens is placed in the common focal plane of the objective and eyepiece lenses. The field lens bends the rays without making them converge or diverge, shifting the exit pupil. If the lens power is chosen such that $1/f_f=1/f_o+1/f_e$ so that the field lens images the objective on the eyepiece then the exit pupil is moved to the eyepiece lens.

Field lens (2)

- In practice it is preferable to place the exit pupil slightly behind the eyepiece lens (easier to put your eye there!)
- Also preferable to place the field lens not quite in the focal plane (1) to permit a graticule to be placed in the focal plane, and (2) to prevent any imperfections in the field lens being imaged by the eyepiece. Compound eyepieces can get very complex!
- Detailed drawing or calculations get quite tricky at this point. Real optical instruments are designed with the help of computer programs, which also use exact calculations rather than making approximations.

Practicalities (4)

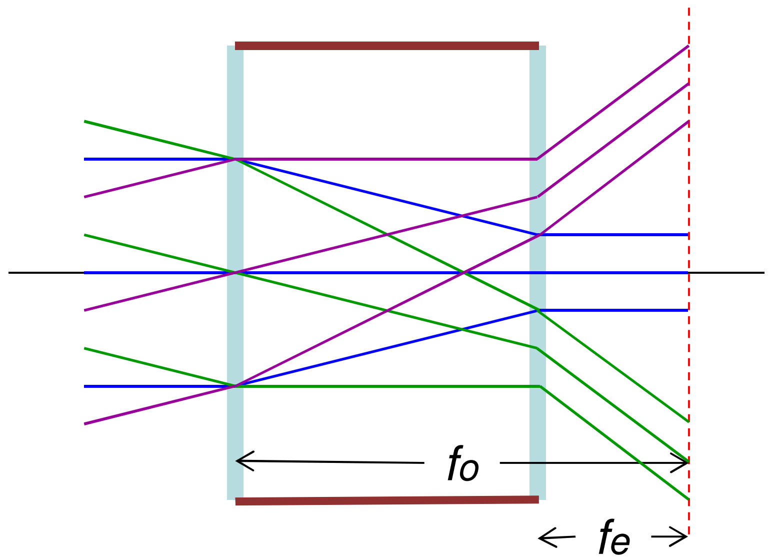
- The effects of aberrations, which arise from break downs in the approximations used, are very important. We will look *briefly* at some of these later.
- The effects of *diffraction,* which limit the resolution achievable with any optical instrument, will be treated in Hilary term. For the moment just note that the bigger the objective lens the better! A traditional rule of thumb for the *practical limit of magnification* is to multiply the objective diameter in inches by 60.
- Distortion by temperature fluctuations in the air is also a major problem for earth based telescopes

Focusing

- In principle you don’t need to focus an astronomical telescope as the object is at a fixed distance (infinity).
- Do need to focus terrestrial telescopes
- Also may need to correct for short sighted observers who need the image closer than infinity

- Object and objective lens are fixed so focus by moving the eyepiece lens towards or away from the objective.

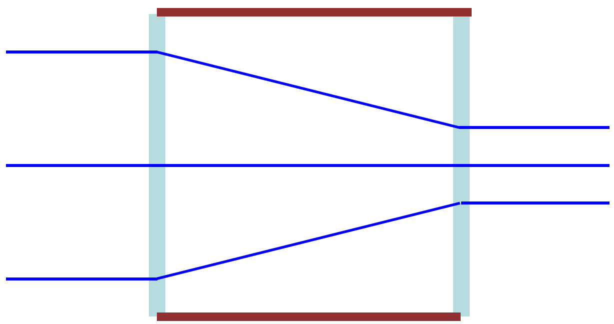
Galilean telescope (1)



The Galilean telescope uses a diverging lens for the eyepiece. The algebra is identical but the eyepiece has a *negative* focal length and so must be placed in front of the focus of the objective, and the magnification is *positive* (image is upright). There is no good pupil position!

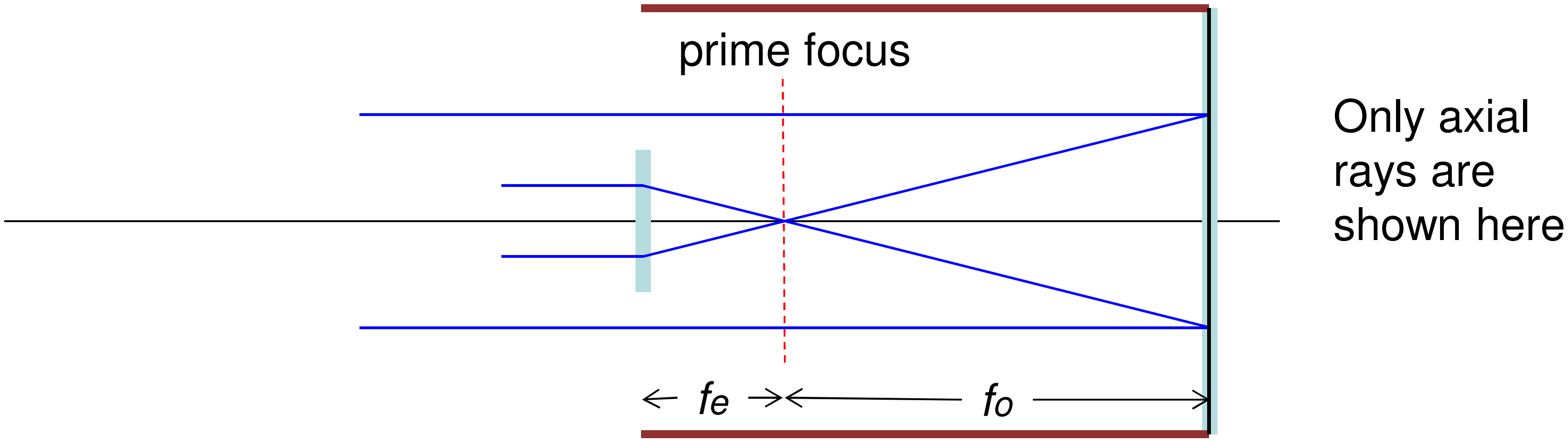
Galilean telescope (2)

- The very poor exit pupil of the Galilean telescope means that it has a very restricted field of view, and so is rarely used as a telescope. It does, however, find use for changing the width of parallel beams of light



- Better than an astronomical telescope for reshaping intense laser beams as it avoids extreme intensities at the focal plane

Reflecting telescopes (1)



A reflecting telescope replaces the objective lens with a converging mirror. The conceptually simplest design puts a detector at the prime focus or an eyepiece just beyond it. More practical designs exist.

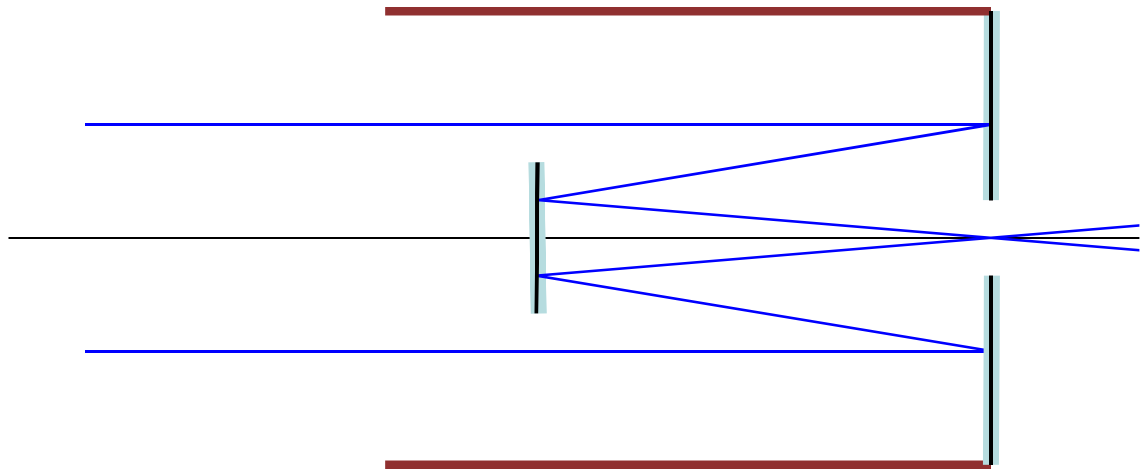
Reflecting telescopes (2)

- Many advantages over refracting telescopes:
  - Large mirrors much easier to make than large lenses
  - No chromatic aberration at main mirror. With prime focus photography there are no lenses and so no chromatic aberration at all
  - Don’t need transparent materials

- Partial blocking of aperture is not ideal but not awful

- Essentially all large telescopes use reflecting designs

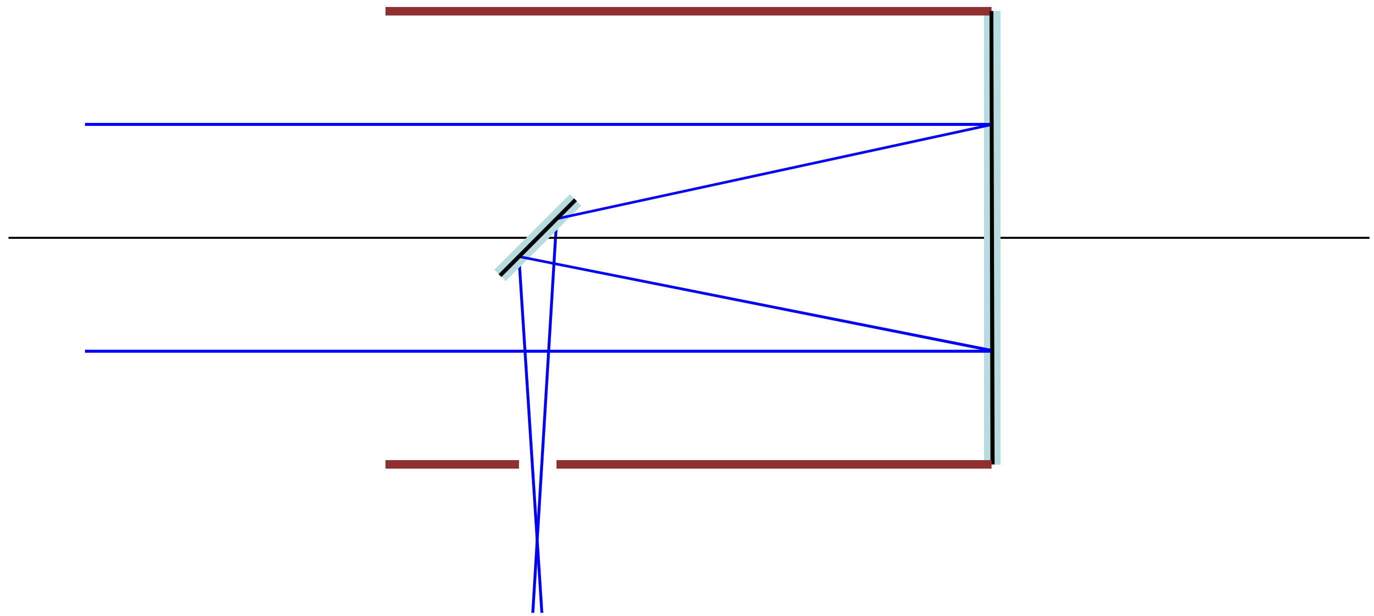
Secondary mirrors



A secondary mirror allowing a longer focal length to be “folded up” in a short tube, and also allowing the detector or eyepiece to be placed conveniently

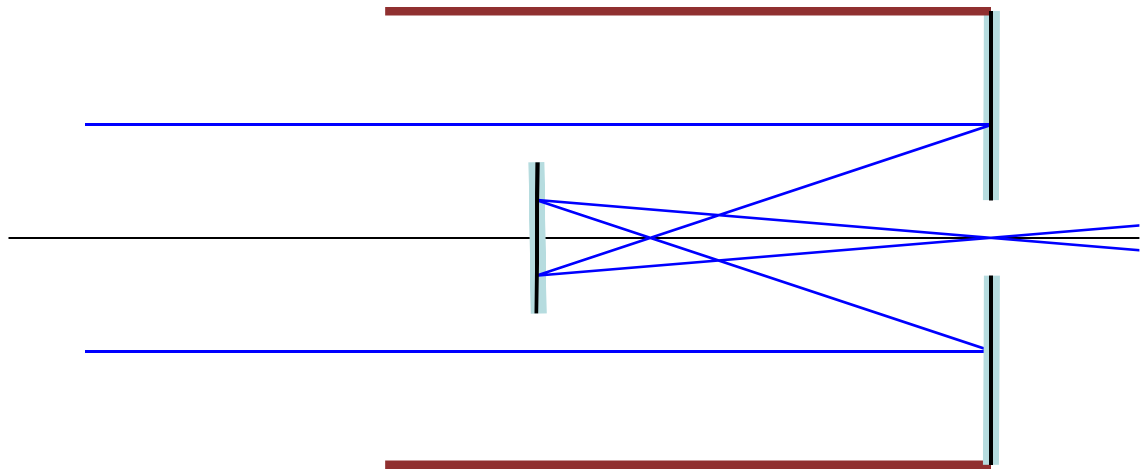
Secondary mirror supports are a minor issue, leading to “spikes” visible around bright stars.

Newtonian telescope



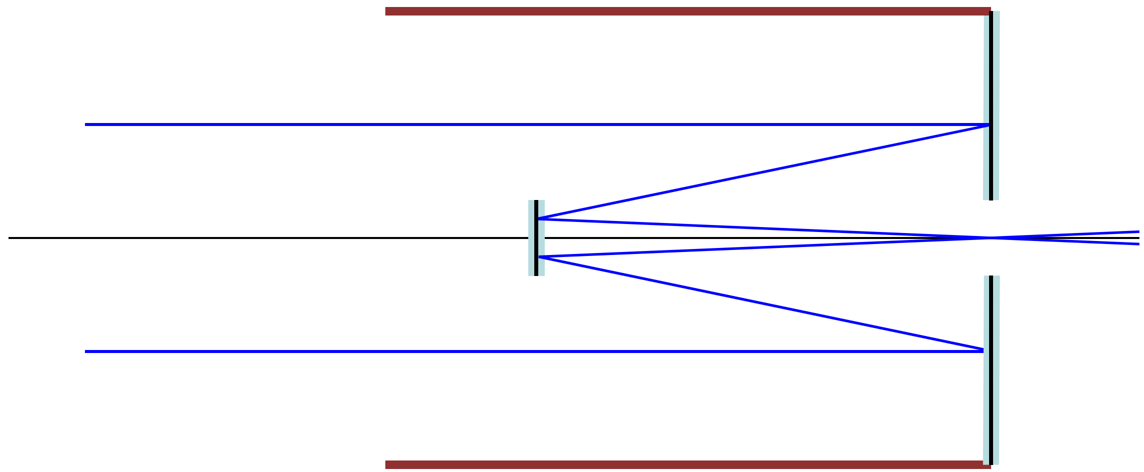
Plane mirror at 45 degrees directs light to a focus on one side of the main telescope tube. The secondary mirror can be quite small in this case

Gregorian telescope



A *converging* secondary mirror is placed after the prime focus. This loses the advantage of "folding up" the optical path, but the second real image has been inverted twice and so is the right way up!

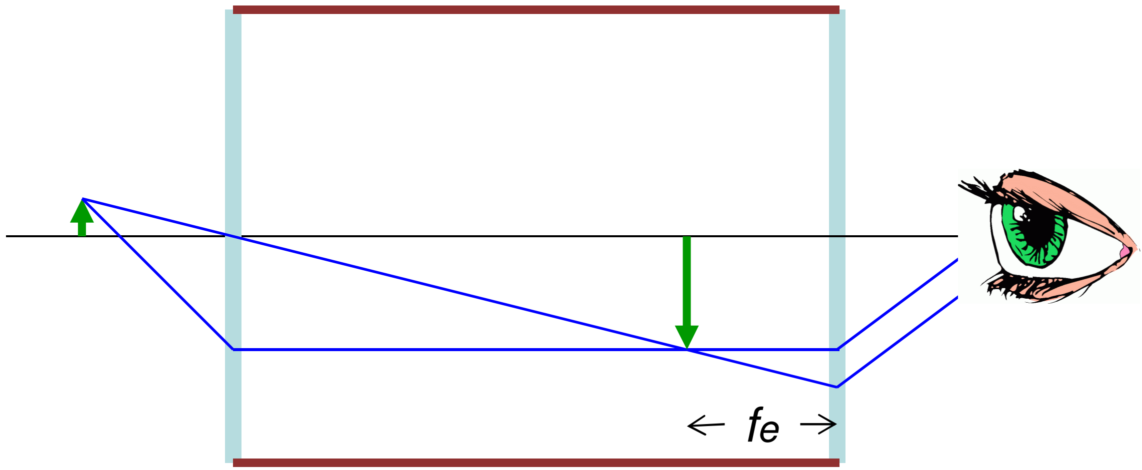
Cassegrain telescope



A diverging secondary mirror acts like a Barlow lens, allowing a *very* long focal length to be "folded up" in a short tube.

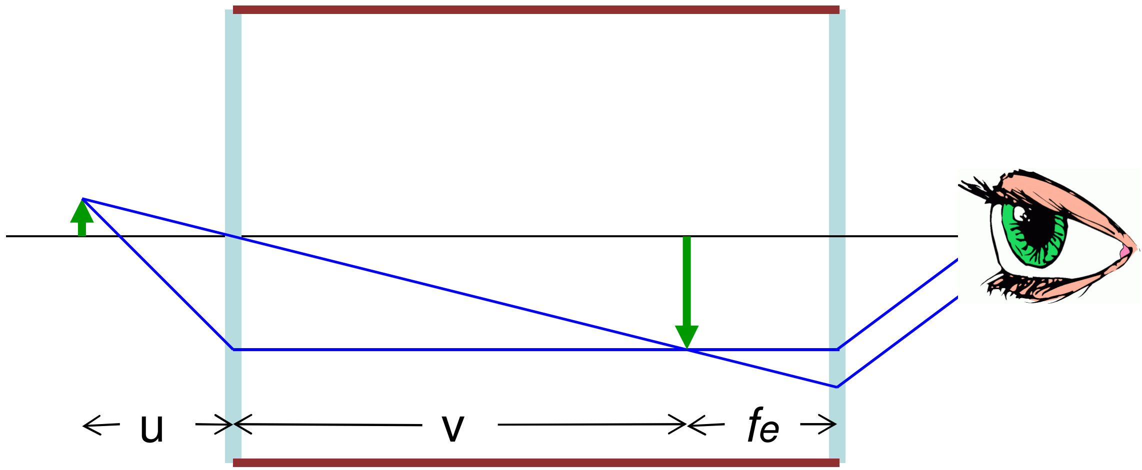
Almost all modern telescopes are variants of the Cassegrain design

Compound microscope (1)



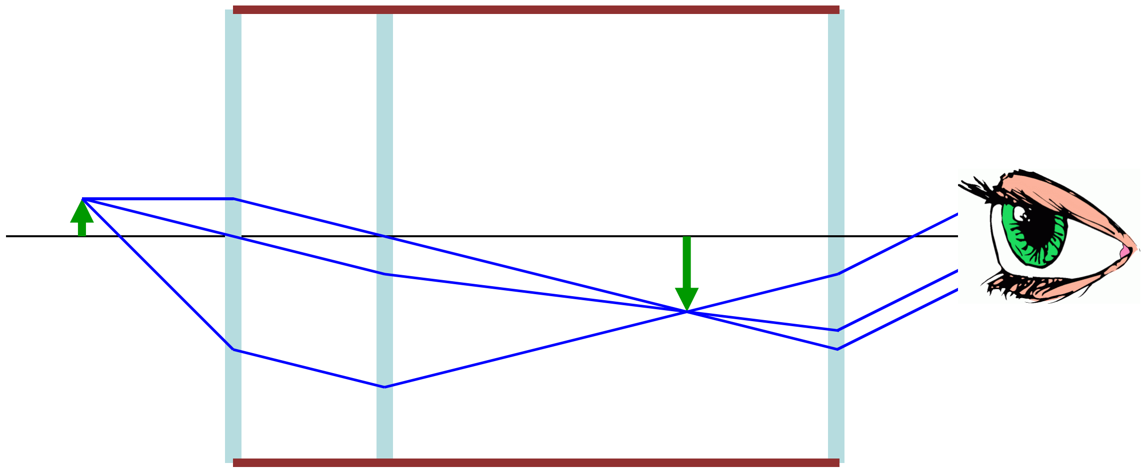
Similar to the astronomical telescope except that the (small) object is placed very near the focus of the objective lens. The objective lens forms an enlarged inverted real image which is observed using the eyepiece as a simple magnifier with the image formed either at infinity (as shown) or at D.

Compound microscope (2)



Total magnification of the system is the product of the magnification achieved by the objective (v/u) multiplied by the magnification achieved by the eyepiece (D/f_e).

Compound microscope (3)



Real microscopes replace the single objective lens with a pair of lenses: a strong objective lens to make a beam of parallel rays and a weak tube lens to form a real image. Magnification of the objective is the ratio of the two focal lengths f_t/f_o. Alternatively the tube and eyepiece lenses form a telescope to observe virtual image formed by objective lens.

Focusing

- In compound microscopes the distance between the tube and eyepiece lenses is usually fixed.
- Moving the objective lens has no effect as the light rays are parallel between the objective and tube lenses.

- Focus a microscope by moving the object with respect to the objective lens (naively to its focal point).

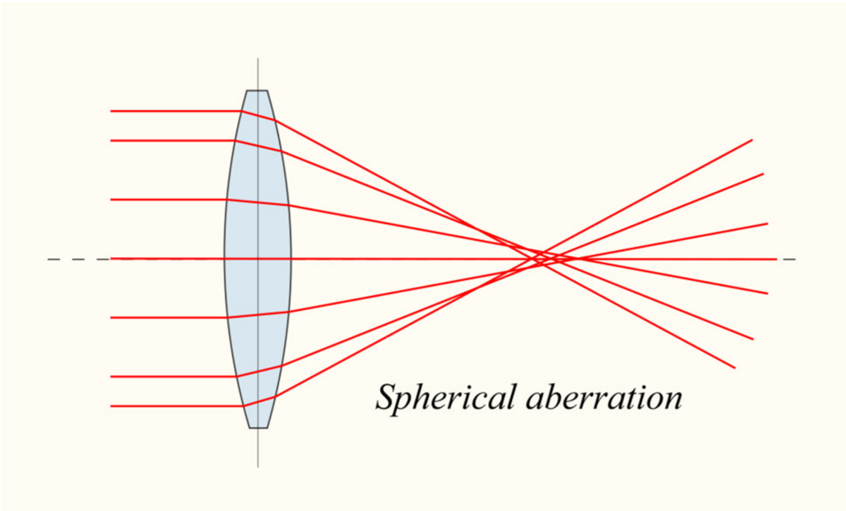
Aberrations

- All optical instruments are subject to aberrations which arise from breakdowns in approximations

- Spherical aberration and coma arise from the breakdown of the paraxial approximation in lenses and mirrors
- Chromatic aberration arises from frequency dispersion (lenses only)
- Many other minor issues! But these are the big three.

- Optical surfaces should be *smooth* to within fractions of a wavelength

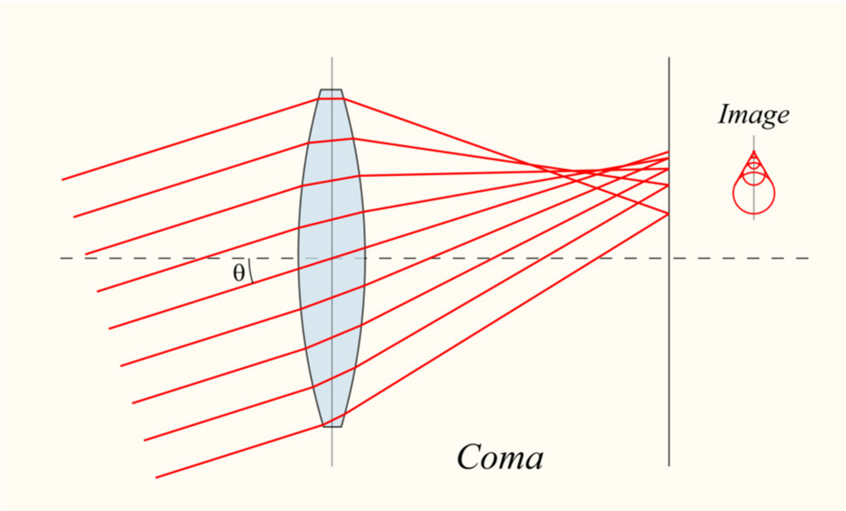
Spherical aberration



Spherical aberration arises because spherical surfaces are not quite the right shape

- Outer parts of lens are *too curved* so focal length becomes shorter at the edges.
- Can be reduced with *aspheric* lenses or mirrors (parabolic mirrors work well).

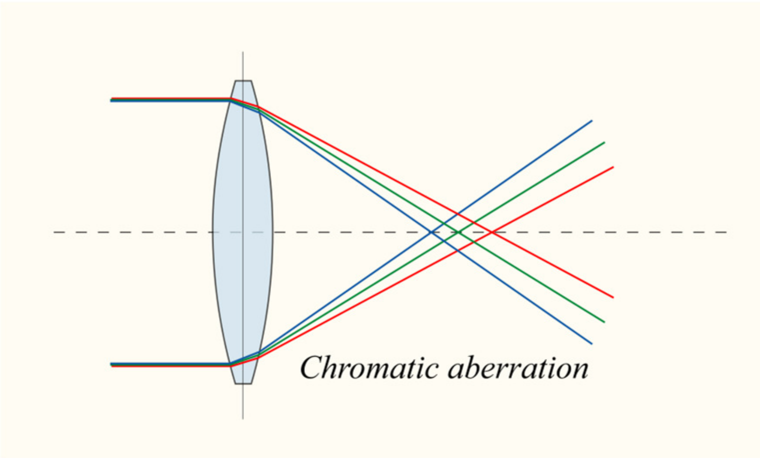
Coma

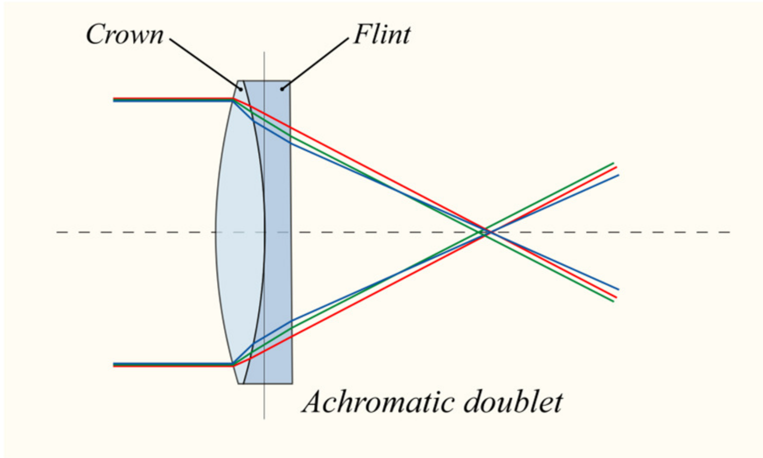


Coma arises from off-axis sources, so image looks worse at the edges

- Can be reduced with aspheric designs
- But parabolic mirrors are very susceptible to coma, so hard to reduce both spherical aberration and coma at the same time
- Multiple elements can compensate each other

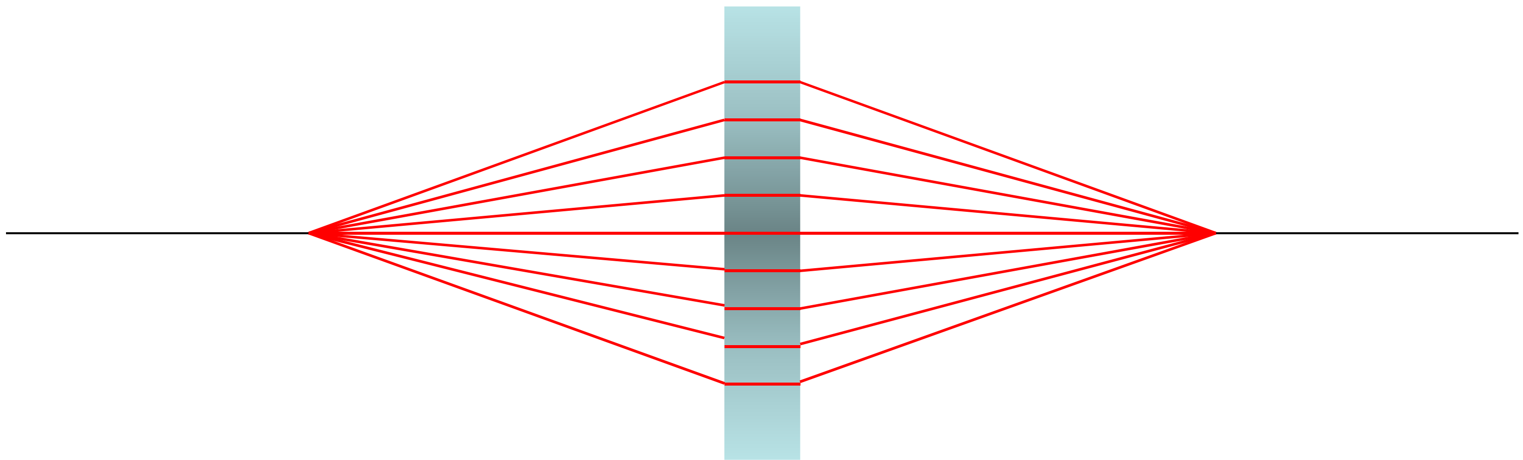
Chromatic aberration





Variation in refractive index means that the focal length of a lens depends on wavelength and so it is impossible to focus all colours to the same point. The *achromatic doublet* uses errors in two lenses to cancel each other, so red and blue light have the same focal length. *Apochromatic* lenses are even better.

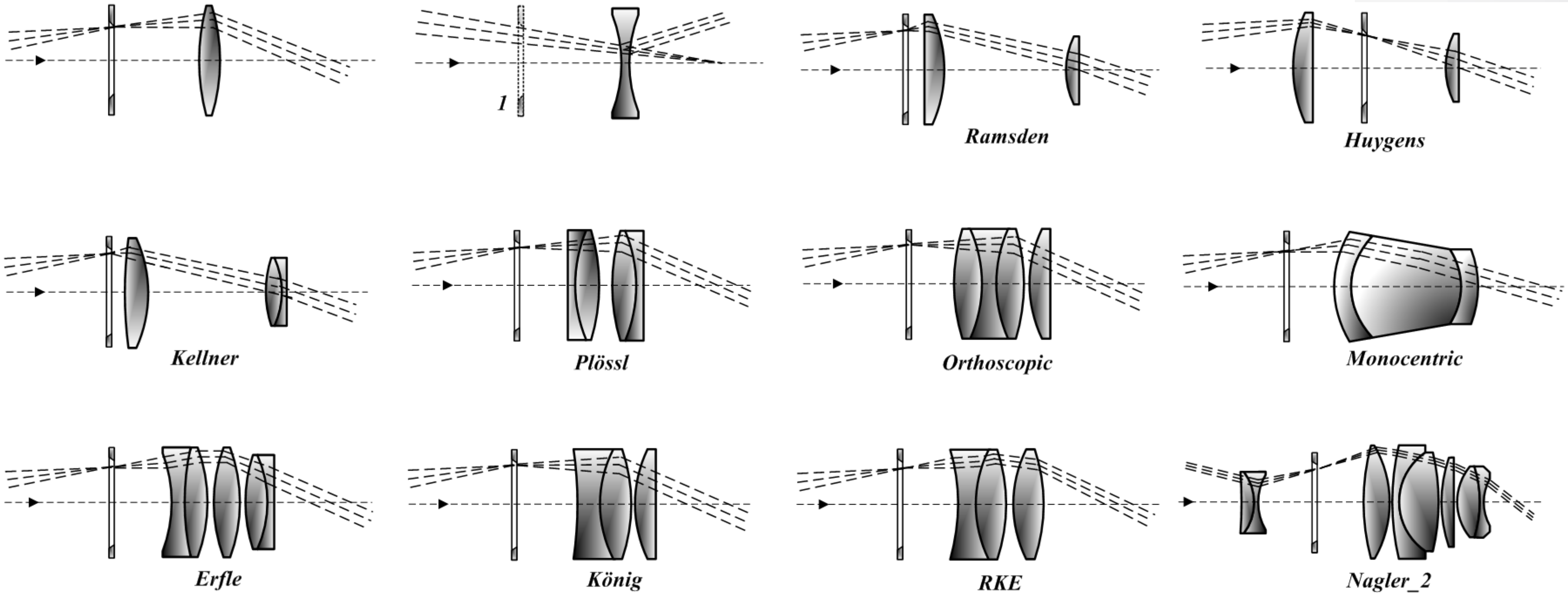
Graded index (GRIN) lenses



- Instead of changing shape use a higher refractive index in the middle. Obvious how they work using Fermat
- Complex to manufacture but they work very well.
- Can combine shaping with GRIN effects
- Natures uses GRIN extensively in animal eyes

Eyepieces

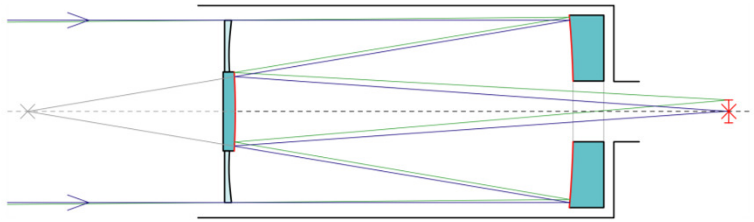
WIKIPEDIA
The Free Encyclopedia



- Modern telescope eyepieces can be very complex! Analysis is well beyond the scope of this course.

Schmidt–Cassegrain

- Cassegrain telescope with spherical mirrors will be vulnerable to spherical aberration
- An aspheric lens is placed at the front to correct this
- Basis of popular designs for “amateur” systems



Philip Wetton telescope

Ritchey–Chrétien telescope

- A Cassegrain design with two hyperbolic mirrors giving low spherical aberration and coma
- Expensive to build and very hard to test
- Standard design for big telescopes such as Hubble